

Regular Wff 'N Proof

Note: Regular Wff 'N Proof uses the seven rules used in Basic Wff 'N Proof (Ko,Ki,Rp,Ai,Co,Eo,Ei) plus these sub-proof rules, Ci, Ao, No, Ni, and Reiteration, (R). Sub-proofs are smaller proofs inside the main proof. Reiteration is the rule that brings a wff from outside a sub-proof into the sub-proof.

Ci: C in. Ci means you are going to make a C wff by putting a smaller wff into a C wff's w2 spot. w1 is free. You are making a C wff, Cw1w2. w1 is supposed(free) and you need to get w2. Most of the time w2 is written in the main proof as a premise. Reiterate it into the subproof.

	q	→	Crq	
1	q		s	
2	a	r	s	
	b	q	1,R	
3	Crq		2,Ci	

Solution: q/R,Ci

	p	→	Csp	
1	p		s	
2	a	s	s	
	b	p	1,R	
3	Csp		2,Ci	

Solution: p/R,Ci

	r	→	Cpr	
1	r		s	
2	a	p	s	
	b	r	1,R	
3	Cpr		2,Ci	

Solution: r/R,Ci

	Ksq	→	Cps	
1	Ksq		s	
2	a	p	s	
	b	Ksq	1,R	
	c	s	b,Ko	
3	Cps		2,Ci	

Solution: Ksq/R,Ko,Ci

Ao: A out means you will take the same wff out of w1 of the A wff and w2 of the A wff. You will have a double sub-proof one after another. Write you're a wff on line 1, first. Put w1 of the A wff at the top of the first sub-proof and w2 at the top of the second sub-proof. Each sub-proof has to end with the same wff. Sometimes you don't get anything from each of these wffs(w1 and w2), but you Reiterate the same wff at the bottom of each sub-proof. I'll explain later about this.

	Arr	→	r
1	Arr		s
2	a	r	s
	b	r	a,Rp
3	a	r	s
	b	r	a,Rp
4	r		2,3,Ao

Solution: Arr/Rp,Ao

	ApKsp	→	p
1	ApKsp		s
2	a	p	s
	b	p	a,Rp
3	a	Ksp	s
	b	p	a,Ko
4	p		2,3,Ao

Solution: ApKsp/Rp,Ko,Ao

	ApKqp	→	p
1	ApKqp		s
2	a	p	s
	b	p	a,Rp
3	a	Kqp	s
	b	p	a,Ko
4	p		2,3,Ao

Solution: ApKqp/Rp,Ko,Ao

	ACsqEsq	→	Csq
1	ACsqEsq		s
2	a	Csq	s
	b	Csq	a,Rp
3	a	Esq	s
	b	Csq	a,Eo
4	Csq		2,3,Ao

Solution: ACsqEsq/Rp,Eo,Ao

Ao: A out by Reiteration. In the four previous examples we made each sub-proof end with the same wff by using a rule in the sub-proof. With these examples we will Reiterate the wff that each sub-proof must end in into the sub-proof. Each sub-proof ends in the same wff.

	Asp,q \longrightarrow q	
1	Asp	s
2	q	s
3	a s	s
	b q	2,R
4	a p	s
	b q	2,R
5	q	3,4,Ao

Solution: Asp,q/R,Ao

	Aqp,r \longrightarrow r	
1	Aqp	s
2	r	s
3	a q	s
	b r	2,R
4	a p	s
	b r	2,R
5	r	3,4,Ao

Solution: Aqp,r/R,Ao

	AEsqs,p \longrightarrow p	
1	AEsqs	s
2	p	s
3	a Esq	s
	b p	2,R
4	a s	s
	b p	2,R
5	p	3,4,Ao

Solution: AEsqs,p/R,Ao

	AqCsp,r \longrightarrow r	
1	AqCsp	s
2	r	s
3	a q	s
	b r	2,R
4	a Csp	s
	b r	2,R
5	r	3,4,Ao

Solution: AqCsp,r/R,Ao

No: N Out means you can prove anything. You have two wffs that are opposites. These are called contradictions. One wff is an N wff and the second is the wff without the N in front. Examples: Nr and r , Ns and s , Np and p , $NAsq$ and Asq , $NKrp$ and Krp , etc. NNs and Ns also qualify because it has one less N. Make a sub-proof. Put an N in front of the goal on step (a) in the sub-proof. Reiterate these two contradictions on step (b) and step (c) and then come out of the sub-proof, take the N off of the step (a) N wff and write it on step 4 with No(N out).

		Ns,s \longrightarrow Erq	
1	Ns	s	
2	s	s	
3	a	NErq	s
	b	Ns	1,R
	c	s	2,R
4	Erq	4,No	

Solution: Ns,S/R,No

		Nr,r \longrightarrow Asp	
1	Nr	s	
2	r	s	
3	a	NAsp	s
	b	Nr	1,R
	c	r	2,R
4	Asp	3,No	

Solution: Nr,r/R,No

		q,Nq \longrightarrow Ksp	
1	q	s	
2	Nq	s	
3	a	NKsp	s
	b	q	1,R
	c	Nq	1,R
4	Ksp	3,No	

Solution: q,Nq/R,No

		KsNs \longrightarrow NEsp	
1	KsNs	s	
2	s	1,Ko	
3	Ns	1,Ko	
4	a	NNEsp	s
	b	s	2,R
	c	Ns	2,R
5	NEsp	4,No	

Solution: KsNs/Ko,R,No

Ni: N in means the same idea as No. You need the contradiction and the sub-proof. The only difference is the goal has to be an N wff in the first place. On step (a) of the sub-proof you take away an N and after you Reiterated the contradiction, you come out of the sub-proof and add the N back in. The rule is Ni.

	$Nq, q \longrightarrow$	$NEsp$
1	Nq	s
2	q	s
3	a Esp	s
	b Nq	1,R
	c q	2,R
4	NEsp	3,Ni

Solution: Nq,q/R,Ni

	$r, Nr \longrightarrow$	$NKsq$
1	r	s
2	Nr	s
3	a Ksq	s
	b r	1,R
	c Nr	2,R
4	NKsq	3,Ni

Solution: r,Nr/R,Ni

	$NAsq, q \longrightarrow$	$NKpr$
1	NAsq	s
2	q	s
3	Asq	2,Ai
4	a Kpr	s
	b NAsq	1,R
	c Asq	3,R
5	NKpr	4,Ni

Solution: NAsq,q/Ai,R,Ni

	$Krs, Nr \longrightarrow$	$NNEsp$
1	Krs	s
2	Nr	s
3	r	1,Ko
4	a NEsp	s
	b Nr	2,R
	c r	3,R
5	NNEsp	4,Ni

Solution: Krs,Nr/Ko,R,Ni