MLAG ADVENTUROUS EQUATIONS® Tournament Rules 2018-19

I. Starting a Match (Round)

A. Two- or three-player matches will be played. A *match* is composed of one or more shakes. A *shake* consists of a roll of the cubes and the play of the game ending with at least one player attempting to write an *Equation*, which contains a mathematical expression that equals the Goal and correctly uses the cubes on the playing mat.

B. The following equipment is needed to play the game:

1. 24 cubes: there are six of each color (red, blue, green, and black). Every face of each cube contains either a digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) or an operation sign (+, –, x, ÷, * or ^, √).

2. A playing mat: this contains four sections.

a. Goal: cubes played here form the Goal.

b. Required: all cubes played here *must* be used in any Solution.

c. Permitted: any or all cubes played here *may* be used in any Solution.

d. Forbidden: *no* cube played here may be used in any Solution.

Comment Many game boards have a section labeled "Resources." However, any reference in these rules to the "playing mat" or "mat" does not include the Resources section.

3. A one-minute sand timer: this is used to enforce time limits.

4. A challenge block: this is a cube or similar object, not a flat object such as a coin. It should not be so large that two players can grab it at the same time.

C. Players may use only pencils or pens, blank paper, and (for Adventurous Equations) variation sheets. No prepared notes, books, tables, calculators, cell phones, or other electronic devices may be used.

In Elementary and Middle Divisions, players may use a preprinted chart for recording the Resources, variations, Goal and Solutions.

D. The Goal-setter for the first shake is determined by lot. On each subsequent shake, the Goal-setter is the player immediately to the *left* of the previous Goal-setter.

To determine the first Goal-setter, each player rolls a red cube. The player rolling the highest digit sets the first Goal. A player who rolls an operation sign is eliminated unless all players roll an operation sign. Players tied for high digit roll again until the tie is broken.

II. Starting a Shake

A. To begin a shake, the Goal-setter rolls all 24 cubes. The symbols on the top faces of the rolled cubes form the *Resources* for the shake.

1. A shake begins as soon as the timing for rolling the cubes is started or the cubes are rolled.

2. During a shake, no player may turn over a cube or obstruct the other players' view of any cube. (See Section IX-C.)

B. In Adventurous Equations, after the cubes are rolled but before the Goal is set, each player must select a variation from the appropriate list in Section **XIII** of these rules. A *variation* is a special rule which, if it conflicts with any of the regular tournament rules, supersedes those rules.

1. The Goal-setter makes the first selection, then the player to the left of the Goal-setter, then the third player if there is one.

a. Each player has 15 seconds to make a variation selection.

b. To begin a shake, the Goal-setter has one minute to roll the cubes. At the end of this minute, he has 15 seconds to select a variation. However, if the Goal-setter selects a variation before the minute for rolling the cubes expires, the next player has the rest of that minute plus 15 seconds to select a variation. If the second player also selects a variation before that minute expires, the third player (if there is one) has the rest of that minute plus 15 seconds to select.

c. A player selects a variation by circling its name in the list for that shake. This list is located on the reverse side of the scoresheet or on a separate sheet. For certain variations (e.g., Base or Multiple of k), the player must also fill in a blank to indicate which base or value of k is chosen, and so on.

2. If a player selects a variation that has no effect on the shake, a variation that conflicts with one already chosen for the shake, or a variation that has already been chosen for the shake, the player loses one point and must pick another variation. If, on the second try, the player still does not select an appropriate variation, he loses another point and may not pick a variation for that shake.

If a player's illegal variation selection is not pointed out before the next player selects a legal variation or a legal Goal is set (whichever comes first), the player making the illegal selection is not penalized. However, the illegal variation is ignored for the shake.

Examples It is illegal to choose 0 Wild when no 0 cube is in Resources or +=Average when no + was rolled.

3. In two-player matches in Elementary, Middle, and Junior Divisions, the player who is not the Goal-setter must select *two* variations for the shake. In Senior Division, any player may pick two variations for any shake in either a two- or three-player match.

A player picking two variations must select both within the 15-second time limit. (See Section **XI- A-1-b**.)

III. Legal Mathematical Expressions

A. A *legal mathematical expression* is one that names a real number and does not contain any symbol or group of symbols that is undefined in Equations.

Example *a* ÷ 0 for any value of *a* does not name a real number. (See Section **C** below for additional examples.)

Comment An expression written on paper may contain pairs of grouping symbols such as parentheses, brackets, or braces even though these do not appear on the cubes. These symbols indicate how the Equation-writer would physically group the cubes if the Equation were actually built with the cubes.

B. The symbols on the cubes have their usual mathematical meanings with the following exceptions:

1. The + and – cubes may be used only for the operations of addition and subtraction; they may not be used as positive or negative signs.

Examples +7, –8, 6x+4, and 17÷ (–8) are illegal expressions.

2. If the radical sign (√) is used, it must always be preceded by an expression to denote its index unless the index equals 2. If no index is shown, it is understood to be 2.

Examples 2√9 or just √9 is legal and means "the square root of 9"; 1√2 means "the first root of 2," which is 2. (2+1)√8 means "the cube root of 8," which is 2. 4x√9 means "4 times the square root of 9," which is 4 x 3 or 12. 3√√9 means "the cube root of the square root of 9," which is the sixth root of 9. (This expression is illegal in Elementary Division – see the "General Rule" in Section **XIII-A**.)

3. * (or ^) means exponentiation (raising to a power). The ^ cube must be written with the point up.

Example 4*2 (or 4^2) means 4^2, which is 4 x 4 =16.

C. Expressions involving powers and roots must satisfy these requirements:

1. Even-indexed radical expressions indicate only *non-negative* roots.

Examples $\sqrt{9}$ equals 3, not –3; $4\sqrt{16} = 2$ (not –2).

2. The following expressions are *undefined*. Note: In all cases, * may be replaced by ^.

a. $0\sqrt{a}$ where *a* is any number

b. $0 * a$ where $a \leq 0$

c. $a \sqrt{} b$ where *a* is an even integer and *b* is negative

d. $(a \div b) \sqrt{} c$ where *c* is negative and, when $a \div b$ is reduced to lowest terms, *a* is an even integer and *b* is an odd integer

e. $a * (b \div c)$ where *a* is negative and, when $b \div c$ is reduced to lowest terms, *b* is an odd integer and *c* is an even integer

Examples

(a) $(-8)^{4/6}$ is defined, as shown by the following steps. First reduce the fractional exponent to lowest terms: $(-8)^{4/6} = (-8)^{2/3}$. $(-8)^{2/3}$ is of the form $a * (b \div c)$ where *a* is negative. Since *b* is even and *c* is odd, $(-8)^{2/3}$ is defined. $(-8)^{2/3} = 3\sqrt{(-8)^2} = 3\sqrt{64} = 4$.

(b) $(-4)^{2/4}$ is *not* defined because $(-4)^{2/4} = (-4)^{1/2}$, which is of the form $a * (b \div c)$ with *a* negative, *b* odd, and *c* even.

Note The following reasoning is *not* allowed since the exponent is not reduced first: $(-4)^{2/4} = 4\sqrt{(-4)^2} = 4\sqrt{16} = 2$.

(c) $^{3/6}\sqrt{(-9)}$ is defined because $^{3/6}\sqrt{(-9)} = {}^{1/2}\sqrt{(-9)}$, which is of the form $(a \div b) \sqrt{} c$, with *c* negative. However, *a* is odd and *b* is even. So $^{3/6}\sqrt{(-9)} = (-9)^{6/3} = (-9)^2 = 81$.

(d) $^{8/2}\sqrt{(-5)}$ is *not* defined because $^{8/2}\sqrt{(-5)} = {}^{4}\sqrt{(-5)}$, which is of the form $a \sqrt{} b$ where *a* is even and *b* is negative.

IV. Setting the Goal

A. The player who rolls the cubes must set a Goal by transferring the cube(s) of the Goal from Resources to the Goal section of the playing mat.

B. A Goal consists of at least one and at most *six* cubes that form a legal expression.

1. Numerals used in the Goal are restricted to one or two digits. The use of operation signs is optional.

Examples of legal Goals 6, 23, 8–9, 17x8, 19+8–5, 87÷13, 3√64, √49
Examples of illegal Goals 125 (three-digit numerals not allowed), 23+18+7 (too many cubes), 45x (does not name a number), +8 (does not name a number since + means addition).

2. The order of operations of mathematics does *not* apply to the Goal. The Goal-setter may physically group the cubes to indicate how it is to be interpreted. If the Goal-setter does not group the cubes, the Goal may be interpreted in any valid way.

Examples

(a) <u>2x 3+5</u> (with space between x and 3) means 2 x (3 + 5).
(b) <u>2x3 +5</u> (with space between 3 and +) means (2 x 3) + 5.
(c) The Goal <u>2x3+5</u> (with no spaces) may be interpreted as either 2 x (3 + 5) or (2 x 3) + 5.

Comment The Goal-setter may not be able to remove all ambiguities from the Goal.

Example <u>√ 5+4 x9</u> where the Goal-setter wants to apply the √ to the entire expression (5+4)x9. *Declaring orally that the √ applies to everything that follows or extending the root over the entire expression in writing is not binding.*

Players may interpret this Goal as [√(5+4)]x9 or as √[(5+4)x9].

3. Once a cube touches the Goal section, it must be used in the Goal.

a. The Goal-setter indicates the Goal has been set by saying “Goal.”

b. The Goal-setter may rearrange or regroup the cubes in the Goal section until he says “Goal.”

c. If the time runs out to set the Goal or the setter turns the timer, it has been set.

d. The Goal may not be changed once it has been set.

C. Before moving the first cube to the Goal section of the mat, the Goal-setter may make a *bonus move*.

1. To make a bonus move, the Goal-setter must say "Bonus," then move one cube from Resources to Forbidden, then move one or more cubes to the Goal.

2. A Goal-setter who is leading in the match may not make a bonus move.

If the Goal-setter makes a bonus move while leading in the match and an opponent points out the error before the next player moves or someone legally challenges, the cube in Forbidden is returned to Resources. The mover also receives a one-point penalty.

D. If the Goal-setter believes no Goal can be set that has at least one correct Solution (see Section **VII**), he may declare "No Goal." Opponents have one minute to agree or disagree with this declaration.

1. If all players agree, that shake is void and the same player repeats as Goal-setter for a new shake.

Comments

(a) The Goal-setter would declare "No Goal" only in those rare instances when an unusual set of Resources was rolled. For example, there are less than three digit cubes or only one or two operation cubes. (Even in these cases, the Goal-setter could choose a variation like 0 Wild that might allow a Goal to be set.)

(b) Players receive no points for the void shake.

(c) If the Goal-setter makes a Bonus move, he is committed to setting a goal and may not declare "No Goal"

2. An opponent who does not agree with the "No Goal" declaration indicates disagreement by picking up the challenge block (see Section **VI-B**) and challenging the "No Goal" declaration. She then has two minutes to write a correct Equation. If there is a third player, he also can choose to write an Equation. The Challenger and Third Party may use as many cubes from Resources as needed for the Goal and Solution. Scoring for a challenged "No Goal" is as follows:.

- **If the Challenger presents a correct Equation, he scores 6. If the Challenger's Equation is incorrect, he scores 2.**
- **If the Third Party presents an incorrect Equation, she scores 2. If the Third Party presents a correct Equation, she scores 4. If the Third Party does not present an Equation, she scores 6 if the Challenger's Equation is incorrect or 2 if the Challenger's Equation is correct.**
- **If either the Challenger or the Third Party presents a correct Equation, the original Goal-setter scores 2. If neither the Challenger nor the Third Party presents a correct Equation, the original Goal-setter scores 6.**

V. Moving Cubes

A. After the Goal has been set, play progresses to the left (clockwise direction).

B. When it is your turn to play, you must either move a cube from Resources to one of the three sections of the playing mat (Required, Permitted, Forbidden) or challenge the last Mover.

The move of a cube is completed when it touches the mat. Once a cube is legally moved to the mat, it stays where it was played for the duration of the shake.

C. If you are not leading in the match, then on your turn you may take a bonus move before making a regular move.

1. To make a bonus move, the Mover must say "Bonus," then move one cube from Resources to Forbidden, then another cube to Forbidden, Permitted or Required.

Comments

(a) If you do not say 'Bonus' before moving the cube to Forbidden, the move does not count as a bonus move but as a regular move to Forbidden. You are not entitled to play a second cube.

(b) When making a bonus move, the first cube *must* go to Forbidden. The second (bonus) cube may be moved to Required, Permitted, or Forbidden.

2. If the player in the lead makes a bonus move and an opponent points out the error before another player makes a legal move or challenge, the Mover must return the second cube played on that turn to Resources. The Mover also receives a one-point penalty.

Comments

(a) Players tied for the lead may make Bonus moves.

(b) Players often call "Bonus" and move two cubes simultaneously to Forbidden. If the player did not call "Bonus", he may return either of the two cubes to Resources.

VI. Challenging

A. Whether or not it is your turn, you may challenge another player who has just completed a move or set the Goal. The two main challenges are Now and Impossible.

Note Players may also challenge a "No Goal" call, see Section IV-D-2.

1. By challenging *Impossible*, a player claims that no correct Equation can be written regardless of how the cubes remaining in Resources may be played.

Comments

(a) If the Goal is not a legal expression, an opponent should challenge Impossible. Examples of such Goals are +8, 65+87–3, 122, and so on.

(b) Occasionally it is obvious before the Goal-setter completes the Goal that no Solution is possible. Examples: Using more than six cubes in the Goal or (in Mid/Jr/Sr) using an 8 or 9 in the Goal when Base eight was called. However, opponents must still wait until the Goal-setter indicates the Goal is finished before challenging. You may not pick up the challenge block and "reserve" the right to challenge when the Goal is completed.

(c) A Player who challenges "Never" will be considered to have challenged "Impossible". There will be no penalty for saying "Never" instead of "Impossible".

2. By challenging *Now*, a player claims that a correct Equation can be written using the cubes on the mat and, if needed, *one* cube from Resources.

a. A player may challenge Now only if there are at least two cubes in Resources.

If a player challenges Now with fewer than two cubes in Resources, the challenge is invalid and is set aside. The player who called Now also receives a one point penalty.

Comment If only one cube remains in Resources and no one challenges Impossible, then a Solution is possible using that one cube. Since the latest Mover had no choice but to play the second-to-last Resource cube to the mat, it is not fair that he be subject to a Now challenge. (However, an Impossible challenge could be made.) See Section **VIII** for the procedure to be followed when one cube remains in Resources.

b. Since a correct Solution must contain at least two cubes, it is illegal to challenge Now after the Goal has been set but before a cube has been played to Required or Permitted. If a player challenges Now before any cubes have been played to Required or Permitted, the challenge is invalid and is set aside. The player who called Now also receives a one point penalty.

B. A challenge block is placed equidistant from all players. To challenge, a player must pick up the block and say "Now" or "Impossible."

If a player picks up the block, then decides not to challenge (without saying "Now" or "Impossible"), the player accepts a one-point penalty and play continues. A player who picks up the block and makes a challenge against himself is also penalized one point, and the challenge is set aside.

Comments

(a) The purpose of the block is to determine who the Challenger is in a shake.

(b) Touching the challenge block has no significance. However, players may not keep a hand, finger, or pencil on, over, or near the block for an extended period of time. (See Section **IX-C**.)

(c) A player must not pick up the challenge block for any reason except to challenge. For example, don't pick it up to say "Goal" or to charge illegal procedure or when fewer than two cubes remain in Resources.

VII. Writing and Checking Equations

A. After a valid challenge, at least one player must present an Equation.

1. After a Now Challenge,

- the Challenger *must* present an Equation.
- the Mover may *not* present an Equation.
- the Third Party in a three-player match *may* present an Equation.

2. After an Impossible Challenge,

- the Challenger may *not* present an Equation.
- the Mover *must* present an Equation.
- the Third Party in a three-player match *may* present an Equation.

B. To be *correct*, a Solution must be a legal expression (see Section **III**) that satisfies the following criteria:

1. The Solution must be part of a complete Equation in this form:

Solution = Goal

Comment While *Solution = Goal* is the *recommended* form for writing the Equation, *Goal = Solution* is acceptable. (See Appendix for all matters involving how Equations are written.)

2. The Solution must equal the interpretation of the Goal that the Equation-writer presents with the Solution.

*Examples (in each case * may be replaced by ^.)*

Goal	Sample Equation	Goal	Sample Equation
37	(6x6) + 1 = 37	11+5	(3x2)+(5x2) = 11+5
3x5+2	(5*2) – 4 + 0 = 3x(5+2)	3x 5+2	(5x4)+1 = 3x(5+2)
03	(5x5) – 2 = 23 (with 0 Wild) ↑ ↑ 0 0	0+3	1*(7+8-4x3) = 0+3 (0 Wild, Upside-down) ↑ usd 2
30x7	[(5*2)x8] + 6 + 4 = 30x7 (with 0 Wild, 0 defaults to 0)	9÷8	5 + 4 = 9! ÷ 8! with Factorial variation

Note: See Appendix for a complete list of ways of indicating what ambiguous cubes (such as wild cubes) represent in Equations. The Appendix also lists the default values of ambiguous symbols if an Equation-writer does not indicate the interpretation. However, there is no default order of operations in Equations (except when certain variations, such as Factorial, are played– see Section **6b** below).

Comments

(a) An Equation-writer who does not write a complete Equation (Solution = Goal), even when there is only one interpretation of the Goal, is automatically incorrect.

(b) The Equation-writer does not write the *value* of the Goal except in those cases where writing the Goal is the same as writing its value.

Examples

(i) The Goal is 37.

(iii) The Goal is 40 with 0 Wild, and the writer writes 45 to indicate what 0 represents.

(iii) For a Goal like 3x5+2, the writer must write either (3x5)+2 or 3x(5+2) and not 17 or 21.

(c) For a Goal like 3x5+2, the writer must write either (3x5)+2 or 3x(5+2) and not 17 or 30.

(d) If the Goal is grouped, as in 3x 5+2, an Equation-writer must write 3x(5+2) and not 3x 5+2 (with space between x and 5 but no parentheses). In the latter case, the Goal is ambiguous and a checker may group it in such a way as to make the Equation wrong.

3. The Solution uses the cubes correctly.

a. The Solution contains at least *two* cubes.
b. The Solution uses *all* the cubes in Required.
c. The Solution uses *no* cube in Forbidden.

Comment "Since several Resource cubes may show the same symbol, it is possible to have a 2 in Forbidden which must *not* be used in the Solution at the same time that there is a 2 in Required which *must* be used."

d. The Solution may use one or more cubes in Permitted.

e. After a Now challenge, the Solution must contain *at most one* cube from Resources.

f. After an Impossible challenge, any cubes in Resources are considered to be in Permitted and therefore may be used in the Solution.

4. The Solution contains only one-digit numerals.

Comment Certain variations (see Section **XIII**) allow exceptions to this rule; for example, Two-digit Numerals in Elementary Division and Base *m* in Middle/Junior/Senior.

5. In Adventurous Equations, the Equation satisfies all conditions imposed by the variations selected for that shake. (See Section **XIII** for a list of the variations.)

Examples

(a) If the Elementary variation Three-operation Solution has been chosen, any Solution that contains fewer than three operations is incorrect.

(b) In Middle, Junior, and Senior, the Multiple of *k* variation requires that any Solution *not* equal the Goal. Instead the Solution must differ from the Goal by a multiple of *k*.

6. The Solution is not ambiguous. An *ambiguous Solution* is one that has more than one legal interpretation. Such a Solution is incorrect if an opponent shows that one of its values does not equal the interpretation of the Goal provided with the Solution.

Comment For the procedure to be followed when an Equation-checker thinks a Solution is ambiguous, see Section **C-5-c** below.

a. In Adventurous Equations, the general order of operations of mathematics (roots/exponents first, then multiplication/division, finally addition/subtraction) does *not* apply to Solutions. Consequently, a Solution may be ambiguous if the writer does not use parentheses (or other symbols of grouping such as brackets or braces) to indicate the order of operations.

b. Certain symbols have a default interpretation as regards grouping, as follows.
(i) The radical sign (√) applies to just the numeral immediately behind it unless grouping symbols are used.

Examples √4+5 means (√4)+5. An opponent may not interpret √4+5 as √(4+5). √(4+5) + 7 means (√9) + 7. An opponent may not interpret √(4+5) +7 as √[(4+5) + 7].

(ii)For the Factorial variation (see Section **XIII** below), ! applies to just the numeral in front of it unless the Equation-writer uses grouping symbols to indicate otherwise.

Examples 4 + 7! means 4 + (7!). An opponent may *not* interpret it as (4 + 7)!. Suppose the Goal is <u>4 + 7</u>. If an Equation-writer wants 4+(7!), just write 4+7! If the writer wants 11!, write (4+7)!

AGLOA *Note*: There are some variations (Exponent, Number of Factors, Smallest Prime, Imaginary Numbers) that have default interpretations in the AGLOA rules, but **DO NOT** have a default interpretation in the MLAG rules.

Note: None of the default interpretations of symbols restricts a player's right to interpret an ungrouped Goal in any acceptable way

c. When the default interpretation for two symbols conflict, the expression is ambiguous, and the Equation-writer must use grouping symbols to remove the ambiguity.

Example *The expression* √9! Is ambiguous because the default interpretation for Factorial, which says the expression means √(9!), conflicts with the default interpretation of √, which specifies the interpretation as $(\sqrt{9})!$.

C. After the time for writing Equations has expired (or when all Equation-writers are ready), each Equation that is presented must be checked for correctness.

1. After a challenge in a three-player match, the Third Party has two minutes to decide whether or not to present an Equation. If the Third-Party indicates her decision before the two minutes expire, she may not retract the decision.

   Comment The Third Party is not obligated to indicate whether he is presenting an Equation before the time limit expires. However, if the Third Party decides to indicate his decision before time expires, he may:

   (a) State whether or not he will present an Equation;
   (b) indicate which party, Mover or Challenger, the Third Party is "joining" (agreeing with) on the challenge. This can be done verbally or by pointing to the party when asked.
   (c) present or not present and Equation when the time limit for writing Equations expires. If the Third Party does not present an Equation, she is assumed to be joining the player who is not writing an Equation..

2. All Equations must be presented before any is checked.
   - **a.** Once a player presents an Equation to the opponent(s), she may make no further corrections or additions even if the time for writing Equations has not expired.
   - **b.** An Equation is considered presented when the paper is out of the hand of the writer.
   - **c.** Each Equation-writer must indicate the Equation to be checked, including the interpretation of the Goal. A writer who forgets to indicate the Equation must do so when asked..

3. Opponents have two minutes to check <u>each</u> Equation. When more than one Equation must be checked, they may be checked in any order. In a three-player match, *both* opponents must check a player's Equation during the *same* two minutes. No other Equation should be checked during this time.

   Comments

   (a) When both players in a two-way match present Equations after the last cube has been moved (see Section **VIII** below), only one Equation should be checked at a time.
   (b) Players must not physically move the cubes in Required, Permitted, and Resources to form the Solution being checked. This causes arguments over where each cube was played.

4. Within the time for checking an Equation, opponents must accept or reject the Equation. A player who claims an opponent's Equation is not correct must give at least one of the following reasons (or cite one of the reasons in **Section VII-B**).
   - **a.** The Equation is NOT in the form *Solution = Goal (or Goal = Solution)*
   - **b.** The Solution DOES NOT equal the interpretation of the Goal that the Equation-writer presents with the Solution.
   - **c.** The Solution DOES NOT use the cubes correctly.
     - **(i)** The Solution contains only cube.
     - **(ii)** The Solution DOES NOT use *all* the cubes in Required.
     - **(iii)** The Solution uses a cube in Forbidden.
     - **(iv)** After a Now challenge, the Solution needs more than *one* cube from Resources.
   - **d.** The Goal has no legal interpretation.

     Examples

     (a) 7 ÷ 0 when 0 is not wild
     (b) A Goal containing more than six cubes or a three-digit number
     (c) Mid/Jr/Sr: 39 in Base eight
     (d) Elem: 3 √ 9
     (e) Elem/Mid: 8 x when Sideways was not chosen
   - **e.** The Solution contains numerals with multiple digits.

Comment Certain variations (see Section **XIII**) allow exceptions to this rule: for example, Two-digit Numerals in Elementary Division and Base *m* in Middle/Junior/Senior.

f. The Equation DOES NOT satisfy all conditions imposed by the variations selected for that shake.

g. The Equation-writer's interpretation of the Goal is not a legal interpretation.

Examples

(a) The writer makes 0 in the Goal equal another numeral when 0 Wild was not chosen.
(b) The writer groups the Goal in an illegal manner:
e.g., the Goal is grouped on the mat as 5x 3+4 and the writer interprets it as (5x3)+4.
(c) With Multiple Operations, the Equation-writer uses an operation cube in the Goal multiple times.
(d) With Red Exponent, the writer interprets the Goal 312 (red 2) as a three-digit numeral.

h. One or both sides of the Equation is ambiguous and may be grouped so that the Solution does not equal the Goal. If an opponent believes there is a legal interpretation of a Solution or Goal that makes the Equation wrong, that opponent should copy the Solution and/or Goal to his own paper and add symbols of grouping to create a wrong interpretation. If the checker is able to do so, the Equation is incorrect. However, each checker has only *one* opportunity to prove ambiguity. If there is a second checker, the checkers may either work together to prove ambiguity or work separately. If working separately, the second checker may simultaneously and independently try to prove ambiguity. When both checkers are ready (or one is ready and the other has nothing to show), follow the same procedure used for checking the original Equation. That is, each attempt at proving ambiguity is checked. If either shows a legal interpretation of the Solution and/or Goal such that the Solution does not equal the Goal, then the original Equation is incorrect. If each attempt at proving ambiguity fails, the Equation must be accepted as correct. That is, once the Equation-writer starts checking the attempt(s) at proving ambiguity, no further objections to the Equation are allowed.

Comments

(a) In the case where the checkers work separately to prove ambiguity, if the time for checking the Equation runs out, either or both checkers may take an additional minute (paying the one-point penalty to do so). If only one checker wishes to take the additional minute, the other checker may make no further changes to his revision of the Equation. If he tries to do so, then he incurs the one-point penalty also.
(b) Just as two players writing Equations after a challenge or forceout may not communicate with each other, so two checkers attempting to prove ambiguity separately may not communicate while doing so.
(c) Each checker working separately has only one opportunity to prove ambiguity. Similarly, checkers working together have just one joint chance to prove ambiguity.
(d) While only one checker is attempting to prove ambiguity, the other checker may continue to check other aspects of the Equation.
(e) If each checker separately trying to prove ambiguity is ready with his revision of the Equation before the time for checking expires, no -1 penalty is enforced during the time the original Equation-writer checks any attempts at proving ambiguity.

*Examples (in each case, * may be replaced by ^.)*

(a) The Solution in 5*2–4+0 = 3x(5+2) is ambiguous. An opponent may rewrite the Solution as 5*(2–4)+0 so that it does not equal 3x(5+2).
(b) The Solution in 2x4–(3+1) = 4 is ambiguous. An opponent may rewrite it as 2x[4–(3+1)] so that it does not equal 4. However, an opponent may *not* rewrite it as 2x[4–(3]+1) since the brackets interfere with a grouping already in the Solution.
(c) The Goal in (6x4)–2 = 7+5x3 is ambiguous. A checker who rewrites the Goal as (7+5)x3 has shown that the Equation is incorrect.
(d) In Elementary Division, an expression like 9√5*9 must be grouped 9√(5*9) because 9√5 is illegal (undefined) in Elementary. So if an Equation-writer does not group 9√5*9, an opponent may group it (9√5)*9 to make the Equation wrong.

Comment Certain variations (such as 0 Wild) allow cubes to be used for other symbols. If a cube stands for anything other than what is on the cube, the Equation-writer must indicate clearly and unambiguously in writing what each such cube represents. See Appendix for a list of suggested ways of doing this. Appendix also lists any default interpretations when players do not write what symbols represent.

i. The Solution does not equal the Equation-writer's interpretation of the Goal.

(i) Checkers must *make an effort* to determine whether the Solution equals the writer's interpretation of the Goal before rejecting the Equation.

(ii) The checker can give a general argument that the Solution does not equal the Goal.

Examples

(a) The Goal is a fraction or an irrational number, but the Solution equals an integer (or vice-versa).

(b) The Solution equals a value greater than 1000 when the Goal is 50x10. That is, the Solution is clearly too big (or too small) even without calculating its exact value.

(iii) One or both of the checkers may ask a judge to determine whether the Solution equals the Goal. However, the checkers will be restricted in two ways:

- *No further objections to the Equation will be allowed* even if the time limit for checking has not expired. Both checkers must agree that there are no other questions (cubes on the mat, parentheses, procedures, etc.), as this is the final question that a judge will answer.
- If the Solution and/or the Goal in the Equation is ambiguous, the judge will answer "Yes, the Solution equals the Goal" when one legal value of the Solution equals a legal value of the Goal since the checkers did not raise the issue of ambiguity. Furthermore, checkers may not make the catch-all objection, "The Solution does not *unambiguously* equal the Goal." General claims of ambiguity are not allowed. The checker must provide a *specific* grouping of either or both sides of the Equation that makes it incorrect. (See **III-A** on page **E3** for the definition of *legal mathematical expression*.)

Example The Equation is: (5+1)! = √10! ÷ 7

The writer has not removed the ambiguity for √10! However, she clearly wants √**(10!÷7)**. Since the Solution equals that interpretation of the Goal, the judge will rule the Equation correct if no opponent raised the ambiguity issue. Even if a checker claims ambiguity, he must group the Goal as (√10)!÷7 or [√(10!)]÷7 in order to prove the Equation incorrect.

j. A symbol or group of symbols is used ambiguously in the Solution or Goal, and one interpretation of the symbol(s) gives a value that makes the Solution not equal the writer's Goal.

Example

Jr/Sr: With Base Twelve, the Solution in 6 + √4 = 5 + 3 is ambiguous because √ can mean root or the digit eleven.

Note: See Appendix for the default meaning of symbols that may be ambiguous. For example, if 0 is wild and the Equation-writer does not indicate what 0 means, 0 equals 0.

k. A variation is applied wrongly or not at all.

Examples of incorrect Equations

(a) With 0 Wild, the Equation uses a 0 for one symbol and another 0 for a different symbol.

(b) Elem/Mid: With Average, the Solution equals the Goal if + is interpreted as addition but not as average.

(c) Mid/Jr/Sr: With Multiple of *k*, the Solution equals the Goal rather than differing from it by a multiple of *k*.

VIII. Last Cube Procedure

A. If one cube remains in Resources, the next Mover must either play that cube to Required or Permitted or challenge Impossible. When the cube has been moved, each player has two minutes to write an Equation.

The last cube in Resources may *not* be moved to Forbidden. If a player does so, any challenge that is made is set aside and the cube is returned to Resources. There is no penalty unless the player's time to move expires. (See Section **XI**.)

B. An opponent may challenge Impossible against the player who moved the last cube from Resources to Required or Permitted provided the challenge is made by the end of the first minute for writing Equations. If the challenge is made, the Mover (and the Third Party, if siding with the Mover) has the rest of the original two minutes to write an Equation.

Comment Any Now challenge with one cube or zero cubes left is invalid, as is any Impossible challenge made after the first minute for writing Equations. In both cases, the challenge is set aside. A player challenging Now in this case receives a one point penalty.

IX. Illegal Procedures

A. Any action that violates a procedural rule is *illegal procedure*. A player charging illegal procedure must specify (within 15 seconds) the exact nature of the illegal procedure.

1. If a move *is* an illegal procedure, the Mover must return any illegally moved cube(s) to their previous position (usually Resources) and, if necessary, make another move.

The Mover must be given at least 10 seconds to make this correction, unless the original move was made after the 10-second countdown (see Section **XI-A-3** below), in which case the time limit rule (Section **XI-A**) is enforced. In general, there is no direct penalty except that the Mover may lose a point if he does not legally complete his turn during the time limit.

Examples of illegal procedures
Moving out of turn, moving two cubes without calling "Bonus" before the first cube touches the mat in Forbidden, moving the last cube in Resources to Forbidden.

2. If the move is *not* an illegal procedure, the cube stands as played.

Comment There is no penalty for erroneously charging illegal procedure. However, see Section **IX-C** if a player does so frequently.

B. An illegal procedure is *insulated* by a legal action (for example, a move or challenge) by another player so that, if the illegal procedure is not corrected before another player takes a legitimate action, it stands as completed.

Example Suppose a player makes an illegal bonus move (two cubes to Required). Before anyone notices the illegal procedure, the next mover moves (or a valid challenge is issued). In this case, the illegal bonus move stays without penalty.

C. Certain forms of behavior interfere with play and annoy or intimidate opponents. If a player is guilty of such conduct, a judge will warn the player to discontinue the offensive behavior. Thereafter during that round or subsequent rounds, if the player again behaves in an offensive manner, the head judge may penalize the player one point for each violation after the warning. Flagrant misconduct or continued misbehavior may cause the player's disqualification for that round or all subsequent rounds. The head judge may even decide to have the other two opponents replay one or more shakes or the entire round because play was so disrupted by the third party. In some cases, the head judge may order the shake replayed by all three players.

Examples This rule applies to constant talking, tapping on the table, humming or singing, loud or rude language, keeping a hand or finger over or next to the challenge block, making numerous false accusations of illegal procedure, and so on. It also includes not playing to win but rather trying only to ruin the perfect scores of one or both opponents (for example, by erroneously challenging Now or Impossible at or near the beginning of each shake so that both opponents will score 5 for the round), saying one variation but circling another, constantly charging illegal procedure erroneously, counting down the 10-second warning in an obnoxious manner, etc.

X. Scoring a Shake

A. After a challenge, a player is *correct* according to the following criteria:

1. That player had to write an Equation and did so correctly.

If the Third Party agrees with the person who must write an Equation, the Third Party must write a correct Equation also.

2. That player did not have to write an Equation (someone else did), and no opponent wrote a correct Equation.

Exception: After a challenge in a three-player match, a player who does not present an Equation for a shake scores 2 if he accepts another player's Equation as correct even if that Equation is subsequently proven wrong by the other checker.

B. After a challenge, points are awarded as follows.

1. Any player who is not correct scores 2.

AGLOA **2.** Any player who is correct scores 6, unless that player is the Third Party agreeing with the Challenger, in which case the score is 4.

C. After the last cube from Resources is moved to the playing mat and no one challenges Impossible, points are awarded as follows:

1. Any player who writes a correct Equation scores 4.

2. Any player who does not write a correct Equation scores 2.

AGLOA **D.** A player who is absent for a shake scores 0 for that shake.

XI. Time Limits

A. Each task a player must complete has a specific time limit (listed below). The one- and two-minute time limits are enforced with the timer. If a player fails to meet a deadline, he loses one point and has one more minute to complete the task. If he is not finished at the end of AGLOA this additional minute, he loses his turn or is not allowed to complete the task.

Note: In Elementary and Middle Divisions, each one-point penalty must be approved by a judge initialing the scoresheet.

1. The time limits are as follows:

a. rolling the cubes	1 minute
b. making a variation selection This time limit does not begin until after the one minute for rolling the cubes.	15 seconds
c. setting the Goal	2 minutes
d. first turn of the player to the left of the Goal-setter	2 minutes
e. all other regular turns (including any bonus moves)	1 minute
f. stating a valid challenge after picking up the challenge block	15 seconds
g. deciding whether to challenge Impossible when no more cubes remain in Resources If the Impossible challenge is made, any time (up to a minute) the Challenger takes deciding to challenge counts as part of the two minutes for writing an Equation.	1 minute
h. writing an Equation During this time, the Third Party (if there is one) must decide whether to present an Equation after a Now or an Impossible challenge. At the end of these two minutes they must present their Equation.	2 minutes
i. deciding whether an opponent's Equation is correct	2 minutes

2. Often a player completes a task before the time limit expires. When sand remains in the timer from the previous time limit, the next player will receive additional time. An opponent timing the next player may either flip or not flip the timer so as to give the opponent the lesser amount of time before the remaining sand runs out and the next time limit can be started.

3. A player who does not complete a task before all the sand runs out for the time limit must be warned that time is up. An opponent must then count down 10 seconds loud enough for the opponent to hear. The one-point penalty for exceeding a time limit may be imposed only if the player does not complete the required task by the end of the countdown.

The countdown must be done at a reasonable pace; for example, “1,010; 1,009; 1,008…,”

An exception to this rule occurs when a player picks up the Challenge Block but does not state a valid challenge within the 15-second time limit. If the player does not wish to challenge, he loses one point and play continues.

B. A round lasts a specified amount of time (usually 30 minutes). After 30 minutes, players are told not to start any more shakes.

Players have five minutes to finish the last shake. After these five minutes, players still involved in a shake in which no challenge has been made and one or more cubes remain in Resources will be told: “Stop, don’t move another cube – this is the end of the round. Each player has two minutes to write a correct Equation that may use any of the cubes remaining in Resources. Any player who presents a correct Equation scores 4 points for that shake; an incorrect Equation scores 2.”

XII. Scoring a Match

A. Each player is awarded points for the match based on the sum of his scores for the shakes played during that match according to the following tables:

Three-Player Matches	Points
First place	6
Two-way tie for first	5
Three-way tie for first	4
Second place	4
Tie for second	3
Third place	2

Two-Player Matches	Points
First place	6
Tie for first	5
Second place	4

B. When a round ends, each player must sign (or initial) the scoresheet and the winner (or one of those tied for first) turns it in. If a player signs or initials a scoresheet on which his score is listed incorrectly and the error was a simple oversight, then, with the agreement of all players, correct the scores.

However, if there is evidence of intent to deceive and the error was not a simple oversight, then do the following:

1. If the error gives the player a lower score, she receives the lower score.

2. If the error gives the player a higher score, she receives 0 for that round.

XIII. Adventurous Variations

Comment See Section **II-B** for the procedure to be followed when selecting variations.

A. Elementary Variations (grade 6 and below)

Note {counting numbers} = {natural numbers} = {positive integers} = {1, 2, 3, 4, ...}
{whole numbers} = { 0, 1, 2, 3, 4, ...}

GENERAL RULE: If * (or ^) is used for raising to a power, both base and exponent must be whole numbers. If √ is used for the root operation, the index must be a counting number, and the base and total value must be whole numbers.

*Examples (in each case, * may be replaced by ^.)*

(a) 3 * 2 is acceptable and equals 9. 0 * 9 equals 0 and 7 * 0 equals 1. However, 2*(1–3), 4*(1÷2), (2–5)*4, and (2÷3)*3 are not legal in Elementary.

(b) 2√9 or just √9 is acceptable and equals 3. 9√0 equals 0. However, √5 and 3√9 are not legal since neither is a whole number. Also 2√(1÷3), (1÷2)√5, and 3√(1–9) are illegal in Elementary.

(c) The legality of √3*4 depends on its grouping. √(3*4) is legal; (√3)*4 is not.

1. <u>Sideways:</u> A cube representing a non-zero number may be used sideways in the Goal or Solution to equal the reciprocal of that number.

Examples 1 + 2 + 2 1 + 2 + 0.5 = 3.5;
1 ÷ 3 = 1 ÷ (1/3) = 1 x 3 = 3

Comment See Appendix for ways to indicate a sideways cube in an Equation.

2. <u>Upside-down:</u> A cube representing a number may be used upside-down in the Goal or Solution to equal the additive inverse of that number.

Examples 6 x 2 = 6 x (–2) = –12. However, 62 is *not* legal for 6 – 2 or 60 + (–2).

Comment See Appendix for ways to indicate an upside-down cube in an Equation.

Note: The Sideways rule and the Upside-down rule may be used on a single-digit numeral at the same time if both variations have been selected, but only in a Solution.

Examples 3 = -⅓; 8 = -⅛
sw *sw*
ud *ud*

3. <u>0 Wild:</u> The 0 cube may represent any numeral on the cubes, but it must represent the same numeral everywhere it occurs (Goal and Solution). Each Equation-writer must specify in writing the interpretation of the 0 cube if it stands for anything other than 0 in the Equation.

Examples

(a) (0 x 6) – 0 = 15, where both 0's stand for 3, is allowed but (0 x 6) – 0 = 14, where the first 0 stands for 3 and the second for 4, is *not* allowed.

(b) (0 x 6) – 0 = 12, where the first 0 stands for 2 and the second for 0, is *not* allowed.

(c) A 0 in the Goal and any 0 in the Solution must equal the same number. So (8 x 5) + 0 equals the Goal 40 if each 0 equals 2. However, (9 x 5) – 0, where this 0 stands for 5, does *not* equal the Goal 40.

4. <u>Factorial:</u> There are two occurrences of the factorial operator (!) available to be used in the Solution and/or the Goal as the Equation-writer chooses to use them. All uses of ! in the Equation must be in writing.

Comments

(a) 5! ("5 factorial") means 5 x 4 x 3 x 2 x 1, which equals 120. *n*! is defined only for whole number values of *n*. 0! is defined as 1.

(b) In the absence of grouping symbols, ! applies to just the numeral in front of it. See Section **VII-B-6** for examples.

Examples

(a) For the Goal 4 x 30, a Solution of 5! is *not* correct since it contains only one cube.
(b) If the Goal is 9 ÷ 8, an Equation-writer may interpret it as 9! ÷ 8!, which is 9. However, the Solution may *not* contain an ! since both allotted factorial signs have been placed in the Goal (unless Multiple Operations has been called – see below).
(c) The Equation 5 x 4 ÷ 0! = 20 is correct since 0! equals 1 as defined in mathematics texts.
(d) The Equation (8 – 5)! + 2 = 4! ÷ 3 is correct since the Goal is (4 x 3 x 2) ÷ 3 = 4 x 2 = 8.
(e) 3!! = (3!)! = (3 x 2)! = 6! = 6 x 5 x 4 x 3 x 2 = 720

5. <u>Multiple Operations:</u> Every operation sign in Required or Permitted may be used many times in any Solution. If the Factorial variation is also chosen for the shake, an unlimited number of factorial operators may be used in each Solution. At most two factorials may be used in the Goal.

Comments
(a) After an Impossible challenge, any operation sign in Resources may be used many times in a Solution. After a Now challenge, if the one cube allowed from Resources is an operation sign, it may be used multiple times.
(b) Players may simply write an operation sign multiple times in Solutions without any additional comment since an operation cube is not being used to represent another symbol.

6. <u>Three-operation Solution:</u> Any Solution must contain at least three operation symbols. The operation symbols are +, –, x, ÷, * (or ^), √ (and ! if Factorial is chosen).

Comment x used for number of factors is an operation symbol, as is an upside-down radical used for percent. However, a * (or ^) used as a decimal point does not count as an operation.

7. <u>Remainder:</u> *A* ·|· *B* (·|· is a sideways ÷) equals the remainder when *A* is divided by *B. A* and *B* are positive integers, and A is less than or equal to 1000.

Examples
(a) 15 ·|· 2 = 1 since 15 divided by 2 gives a quotient of 7 with remainder 1.
(b) 89 ·|· 15 = 14, the remainder when 89 is divided by 15.
(c) 45·|· 70 = 45 [In general, *A* ·|· *B* = *A* when *A* < *B*.]
(d) 87·|· 87 = 0
(e) 54 ·|· 10 = 4 [In general, *A* ·|· 10 = the last digit of *A*].

The following odd-year variations will be played in Elementary in 2018-19.

8. <u>Two-digit Numerals:</u> Two-digit numerals are allowed in Solutions.

9. <u>LCM:</u> √ may represent the LCM (least common multiple) of two counting numbers.
Comment This variation does not rule out using √ for root, so each Equation-writer must indicate *in writing* which √ in the Equation represents LCM. (See Appendix.)

Examples
(a) 6 √ 8 = 24, the smallest integer divisible by both 6 and 8.
(b) (2 x 3) √ (5 + 4) = 6 √ 9 = 18.
(c) 2 √ 4 = 4 (or 2, the square root of 4).
(d) 0 √ 5 is undefined.
(e) 6√√9 means the LCM of 6 and the square root of 9; that is, the LCM of 6 and 3, which is 6.

10. <u>GCF:</u> * (or ^) may represent the GCF (greatest common factor) of two whole numbers, provided at least one of them is not 0.

GCF(*A*, *B*), "the greatest common factor of *A* and *B*," is defined if *A* and *B* are counting numbers or if *A* is a counting number and *B* = 0 or if *B* is a counting number and *A* = 0. GCF(*A*, 0) = *A* and GCF(0, *B*) = *B*.

Comment This variation does not rule out using * (or ^) for exponentiation. So each Equation-writer must indicate in writing which * (or ^) in the Equation represents GCF. (See Appendix.)

*Examples (in each case, * may be replaced by ^.)*
(a) 8 * 6 = either 8^6 or 2, the largest integer that is a divisor of both 8 and 6.
(b) (4 x 2) * (6 + 3) = 1 (with GCF) or 8^9 (with exponentiation).

11. <u>Number of Factors:</u> x*A* means "the number of counting number factors of *A*," where *A* is a counting number less than or equal to 200.

Comments

(a) This variation does not rule out using x for multiplication. In the Goal or Solution, the meaning of an x cube will usually be clear from the context since number of factors is a unary operation and multiplication is a binary operation.

Examples

(a) x(6 x 2) = 6 (since 12 has six factors: 1, 2, 3, 4, 6, 12)
(b) x(4 x 4) = 5 (since the factors of 16 are 1, 2, 4, 8, 16)
(c) x12 = 6 (for use in any Goal or, if the Two-digit Numerals variation is chosen, in a Solution)
(d) x0, x(1 ÷ 2), x(1 – 4), and x(5 * 4) [or x(5 ^ 4)] are not defined.
(e) xx12 = x(x12) = x6 = 4
(f) In the expression 3xx7, the first x means multiplication and the second means number of factors. 3xx7 = 3 x (x7) = 3 x 2 = 6.
(g) In the expression x4x2, the first x means number of factors and the second means multiplication. By default, the value of this expression is (x4)x2 = 3 x 2 = 6. To obtain the number of factors of 8, the Equation-writer must write x(4x2).

The following even-year variations will be played again in 2019-20.

12. <u>Next Prime Number:</u> x*A* means "the next prime number bigger than *A*," where *A* is a rational number less than or equal to 200.

Comment This variation does not rule out using x for multiplication. In the Goal or Solution, the meaning of an x cube will usually be clear from the context since next prime is a unary operation and multiplication is a binary operation. For example, the Goal 4xx6 has only one interpretation: 4 x (x6), which is 4 x 7 = 28.

(a) x7 = 11, the next prime number bigger than 7.
(b) x(9 ÷ 2) = 5, the next prime bigger than 4.5.
(c) x(0 – 3) = 2, the next prime number bigger than –3. (Note: 1 is not prime.)
(d) xx5 = x(x5) = x7 = 11.
(e) x√49 = x7 = 11. x√67 is undefined because √67 is not a rational number.
(f) In the expression 2xx5, the first x means multiplication and the second means next prime number. The value of this expression is 2 x (x5) = 2 x 7 = 14.
(g) In the expression x5x7, the first x means next prime number and the second means multiplication. The value of this expression is either x(5x7) = 37, the next prime number bigger than 35, or (x5)x7 = 7 x 7 = 49.
(h) There is no limit to the number of consecutive x's in an expression, especially with the Multiple Operations variation also in effect. Thus xxx9 = xx(x9) = x(x11) = x13 = 17.

13. <u>Percent:</u> (upside-down radical) means "percent of." That is, *A* *B* = *A*% of *B* where *A* and *B* are numbers. In the Goal or Solution, *A* and/or *B* may be a two-digit numeral.

Examples

(a) 25 16 = 25% of 16 = 0.25 x 16 = 4.
(b) 6 8 = 6% of 8 = 0.06 x 8 = 0.48.
(c) In a Solution, (8 – 3) (4 + 2) = 5% of 6 = 0.05 x 6 = 0.3.
(d) If the Decimal Point variation (see below) is also in effect, an expression like 1.5 .25 is legitimate in a Solution and equals 1.5% of 0.25 = 0.015 x 0.25 = 0.00375. Similarly, in a Solution, 25 (1.5 x 2) = 25% of 3 = 0.75. And 12.5 16 = 2.

14. <u>Decimal Point:</u> * (or ^) may represent a decimal point. If so used in the Goal or Solution, an * (or ^) may be combined with at most *three* digits to form a numeral. When used as a decimal, * (or ^) takes precedence over all other operations.

Comment This variation does not rule out using * (or ^) for exponentiation. Therefore, Equation-writers are encouraged to write a decimal point instead of * (or ^) when they want to use * (or ^) as a decimal point. Also one * (or ^) may be used as a decimal point and another for exponentiation. If * (or ^) is used as a decimal point, this must be indicated in writing in the Equation. (See Appendix.)

*Examples (in each case, * may be replaced by ^.)*

(a) 2*5 means either 2.5 or 2^5; 3*0 means either 3.0 or 3^0. The Equation-writer must indicate whether the decimal interpretation is desired. One way to do this is to write 2.5 or 3.0 rather than 2*5 or 3*0.

(b) 2*4x2 = 2.4 x 2 or 2^8 or 2^4 x 2; it does *not* mean 2.8. That is, it may *not* be grouped as 2.(4x2). Similarly 4*3! may *not* be interpreted as 4.(3!) or 4.6.

(c) 12*5 means either 12.5 or (in the Goal only unless the Two-digit Numerals variation has been chosen) 12^5; 1*25 means 1.25 or (in the Goal only unless the Two-digit Numerals variation has been chosen) 1^{25}.

(d) 15*0 means either 15.0 or (in the Goal only unless the Two-digit Numerals variation has been chosen) 15^0.

(e) In the Goal or Solution, 255* means 255 and *050 means 0.05.

(f) 15*25 = 15^{25} (in the Goal only unless the Two-digit Numerals variation has been chosen) but has no legitimate interpretation as a decimal since * is used with *four* digits.

(g) 122*5, 1*225, *1225, and 1225* have no interpretations as decimals or powers.

(h) The expression *37*5 may *not* be interpreted as 0.37 1/2 and has no defined interpretation in Elementary Division.

(i) The “digits” are the symbols 0,1,2,3,4,5,6,7,8 and 9. A digit turned sideways or upside-down is no longer a digit. Therefore, with Sideways or Upside-down in effect, you may *not* use an expression like * , *, * or * and interpret the * as a decimal point.

15. <u>+=Average:</u> + shall not represent addition; instead, it shall represent the operation of averaging *two* numbers.

Examples

(a) 7 + 9 = 8, the average of 7 and 9. 7 – (0 – 9) equals 16, as usual.

(b) 5 + (4 x 2) = the average of 5 and 8 = 6.5.

(c) 4+6+9 has two values: (4+6)+9 = 5+9 = 7; 4+(6+9) = 4+7.5 = 5.75. Notice that neither answer equals 19/3, the usual (mathematical) average of 4, 6, and 9.

B. Middle Division Variations (grades 7-8)

1. <u>Sideways:</u> A cube representing a non-zero number may be used sideways in the Goal or Solution to equal the reciprocal of that number.

Examples 1 + 2 + = 1+2 + 0.5 = 3.5;

1 ÷ = 1 ÷ (1/3) = 1 x 3 = 3

Comment See Appendix for ways to indicate a sideways cube in an Equation.

2. <u>Upside-down:</u> A cube representing a number may be used upside-down in the Goal or Solution to equal the additive inverse of that number.

Examples 6 x = 6 x (–2) = –12. However, 6 is *not* legal for 6 – 2 or 60 + (–2).

Comment See Appendix for ways to indicate an upside-down cube in an Equation.

Note: The Sideways rule and the Upside-down rule may be used on a single-digit numeral at the same time if both variations have been selected, but only in a Solution.

Examples $3_{sw\,ud}$ = -⅓; $8_{sw\,ud}$ = -⅛

3. <u>0 Wild:</u> The 0 cube may represent any *symbol* (numeral or operation) on the cubes, but it must represent the same symbol everywhere it occurs (Goal and Solution). Each Equation-writer must specify in writing the interpretation of the 0 cube if it stands for anything other than 0 in the Equation.

Examples

(a) (0 x 6) – 0 = 15, where both 0’s stand for 3, is allowed but (0 x 6) – 0 = 14, where the first 0 stands for 3 and the second for 4, is *not* allowed.

(b) (0 x 6) – 0 = 12, where the first 0 stands for 2 and the second for 0, is *not* allowed.

(c) A 0 in the Goal and any 0 in the Solution must equal the same number. So (8 x 5) + 0 equals the Goal

40 if each 0 equals 2. However, (9 x 5) – 0, where this 0 stands for 5, does *not* equal the Goal 40.

Comments

(a) If a player interprets 0 in the Goal as x, then any 0 in that player's Solution must also be an x.

(b) If 0 Wild and Multiple Operations are both chosen, 0 may be used multiple times in a Solution only if it stands for an operation sign, not a numeral.

(c) If Base eight is also chosen (see below), 0 may not represent the digits "8" or "9". If Base nine is chosen, 0 may not represent "9".

4. Factorial: There are two occurrences of the factorial operator (!) available to be used in the Solution and/or the Goal as the Equation-writer chooses to use them. All uses of ! in the Equation must be in writing.

Comments

(a) 5! ("5 factorial") means 5 x 4 x 3 x 2 x 1, which equals 120. *n*! is defined only for whole number values of *n*. 0! is defined as 1.

(b) In the absence of grouping symbols, ! applies to just the numeral in front of it. See Section **VII-B-6** for examples.

Examples

(a) For the Goal 4 x 30, a Solution of 5! is *not* correct since it contains only one cube.

(b) If the Goal is 9 ÷ 8, an Equation-writer may interpret it as 9! ÷ 8!, which is 9. However, the Solution may *not* contain an ! since both allotted factorial signs have been placed in the Goal (unless Multiple Operations has been called – see below).

(c) The Equation 5 x 4 ÷ 0! = 20 is correct since 0! equals 1 as defined in mathematics texts.

(d) The Equation (8 – 5)! + 2 = 4! ÷ 3 is correct since the Goal is (4 x 3 x 2) ÷ 3 = 4 x 2 = 8.

(e) 3!! = (3!)! = (3 x 2)! = 6! = 6 x 5 x 4 x 3 x 2 = 720

5. Multiple Operations: Every operation sign in Required, Permitted, or Resources may be used many times in any Solution. If the Factorial variation is also chosen for the shake, an unlimited number of factorial operators may be used in each Solution. At most two factorials may be used in the Goal.

Comments

(a) After an Impossible challenge, any operation sign in Resources may be used many times in a Solution. After a Now challenge, if the one cube allowed from Resources is an operation sign, it may be used multiple times.

(b) Players may simply write an operation sign multiple times in Solutions without any additional comment since an operation cube is not being used to represent another symbol.

6. Base *m*: Both the Goal and the Solution must be interpreted as base *m* expressions, where the player choosing this variation specifies *m* for the shake as eight, nine, or ten. Two-digit numerals are allowed in Solutions.

*Examples (in each case, * may be replaced by ^.)*

(a) For Base eight, 37 + 5 = 6 * 2 is a correct Equation. Any Solution or Goal containing the digit "8" or "9" is an illegal expression.

(b) For Base nine, 34 + 5 = 6 * 2 is a correct Equation. Any Solution or Goal containing the digit "9" is an illegal expression.

(c) In Base eight, a Goal like 3 + 8 or 39 should be challenged Impossible. A Goal like 39 is also illegal in Base nine.

7. Multiple of *k*: A Solution must not equal the Goal but must differ from the Goal by a non-zero multiple of *k*, where the player choosing this variation specifies *k* for the shake as a whole number from six to eleven, inclusive. The Goal must not be greater than 1000(in Base 10) or less than –1000 (in Base 10).

Example If *k* = 6 and the Goal is 5, then a Solution must equal 11, 17, 23, 29, and so on, or –1, –7, –13, –19, and so on. A Solution equal to 5 is incorrect.

Comment Multiple of *k* does not require any special writing of the Goal by an Equation-writer. As always, write the interpretation of the Goal, indicating wild cubes and grouping. You do not have to indicate the multiple of *k* difference.

The following odd-year variations will be played in Middle in 2018-19:

8. Powers of the Base: 1 (one) may represent any integral power of ten. (If 1 is used in a two-digit numeral, it stands for 1.) If Base *m* is also chosen, 1 represents any integral power of *m*.

Examples

(a) For Base ten, 9 + 1 may be interpreted as 9 + 1 (since $10^0 = 1$), 9 + 10, 9 + 100, 9 + 1000, and so on, or as 9 + 0.1 (since $10^{-1} = 0.1$), 9 + 0.01, 9 + 0.001, and so on.

(b) If Base eight is chosen, then 1 may represent one, eight, sixty-four, and so on, or one-eighth, one-sixty-fourth, and so on. For Base nine, 1 represents one, nine, eighty-one, one-ninth, etc.

9. Number of Factors: x*A* means "the number of counting number factors of *A*," where *A* is a counting number and *A* is less than or equal to 1000.

Comments

(a) "1000" in the statement of the variation refers to the number one-thousand in base ten. The limit is not reinterpreted if Base Eight or Nine is chosen.

(b) This variation does not rule out using x for multiplication. In the Goal or Solution, the meaning of an x cube will usually be clear from the context since number of factors is a unary operation and multiplication is a binary operation.

(c) If 0 Wild is chosen along with Number of Factors, one 0 may represent number of factors while another 0 may be multiplication since 0 is the symbol x in both cases.

Examples

(a) x(6 x 2) = 6 (since 12 has six factors: 1, 2, 3, 4, 6, 12)
(b) x(4 x 4) = 5 (since the factors of 16 are 1, 2, 4, 8, 16)
(c) x12 = 6 (for use in any Goal or, if the Two-digit Numerals variation is chosen, in a Solution)
(d) x0, x(1 ÷ 2), x(1 – 4), and x(5 * 4) [or x(5 ^ 4)] are not defined.
(e) xx12 = x(x12) = x6 = 4
(f) In the expression 3xx7, the first x means multiplication and the second means number of factors. 3xx7 = 3 x (x7) = 3 x 2 = 6.
(g) In the expression x4x2, the first x means number of factors and the second means multiplication. By default, the value of this expression is (x4)x2 = 3 x 2 = 6. To obtain the number of factors of 8, the Equation-writer must write x(4x2).

10. Any Color Exponent: Any numeral on a _cube may be used as an exponent without being accompanied by an * (or ^) cube. The player selecting this variation chooses a color: red, blue, green, or black and should announce "Red Exponent" or "Blue Exponent", etc. to indicate the color.

Examples

(a) If the chosen exponent color is red, the Goal 253, where the 3 is red, must mean 25^3 since three-digit numerals are illegal.

(b) If blue is the chosen color, a Goal or a Solution like 5^2, where the 2 is on a blue cube, is legal.

(c) If red is the chosen color, an expression like 523 [2 and 3 red] must be interpreted as either 52^3, or $(5^2)^3$, which is 5^6. It may *not* be interpreted as 5*(2*3) or 5^8, because the 2 by itself is no longer an exponent of the 5.

Comments

(a) If Factorial is also chosen, a ! may be placed behind an exponent. So with red exponent, a Goal of 23 (red 3) may be interpreted as $2^{3!}$ or 2^6.

(b) If a player selects Exponent when no digits of the selected color were rolled, that player is penalized one point and must pick another variation.

11. AB+: The Goal and/or the Solution may be or may include a three-cube expression of the form AB+, which is interpreted as a repeating decimal. It may be interpreted as .ABABAB... or as .ABBBBB... A player who presents an Equation must specify in writing which interpretation is used in the Goal and Solution. No decimal points may be used in the Equation (except when the decimal point variation is also chosen for the shake). When the form AB+ is used in an Equation, the Equation-writer should indicate in writing which fraction the AB+ is used for (.ABAB or .ABBB).

Examples

(a) 33+ = 0.333333…(.ABAB interpretation, which is 33/99 and simplifies to 1/3) or 0.333333 (.ABBB interpretation, which also simplifies to 1/3).
(b) 90+ = 0.909090…(.ABAB interpretation, which is 90/99 and simplifies to 10/11) or 0.900000 (.ABBB interpretation, which is [90-9]/90, which simplifies to 9/10).
(c) 1-(55+) = (99/99) – (55/99) = 44/99 (which simplifies to 4/9 in either interpretation).

Comments

(a) If Base *m* is also chosen, the decimal places also change to the appropriate base *m* expression, meaning the fractions change to AB/77 for Base 8, AB/88 for Base 9, etc.
(b) In Base *m*, the numerators and denominators in fractions also must be converted to the appropriate base.

The following even-year variations will be played again in 2019-20.

12. Percent: _/^ (upside-down radical) means "percent of." That is, *A*_/^ *B* = *A*% of *B*, where *A* and *B* are numbers. In the Goal or Solution, *A* and/or *B* may be a two-digit numeral.

If Base *m* and Percent are both chosen, the meaning of percent ("per 100") changes with the base. "Percent" means "per sixty-four" for Base eight and "per eighty-one" for Base nine.

Examples

(a) 25_/^16 = 25% of 16 = 0.25 x 16 = 4.

(b) 6 _/^8 = 6% of 8 = 0.06 x 8 = 0.48.

(c) In a Solution, (8 – 3)_/^ (4 + 2) = 5% of 6 = 0.05 x 6 = 0.3.

(d) If the Decimal Point variation (see below) is also in effect, an expression like 1.5 _/^.25 is legitimate in a Solution and equals 1.5% of 0.25 = 0.015 x 0.25 = 0.00375. Similarly, in a Solution, 25_/^(1.5 x 2) = 25% of 3 = 0.75. And 12.5_/^16 = 2.

(e) In Base eight, 60 _/^11 = $(60_{eight} \div 100_{eight}) \times 11_{eight} = (48_{ten} \div 64_{ten}) \times 9_{ten} = (3_{ten} \div 4_{ten}) \times 9_{ten} = 27_{ten} \div 4_{ten} = (6 + 3/4)_{ten} = (6 + 6/8)_{ten} = 6.6_{eight}$

13. Decimal Point: * (or ^) may represent a decimal point. If so used in the Goal or Solution, an * (or ^) may be combined with at most *three* digits to form a numeral. When used as a decimal, * takes precedence over all other operations.

Example A Goal of 4**5 (or 4^^5) can equal either $4.^5 = 1028$ or $4^{.5} = \sqrt{4} = 2$.

Comment If 0 Wild and Decimal Point are both chosen, 0 may represent a decimal point. Also, one 0 in the Equation may be decimal point and another 0 may be exponentiation since 0 is the symbol * in both cases.

14. +=Average: + shall not represent addition; instead, it shall represent the operation of averaging *two* numbers.

Examples

(a) 7 + 9 = 8, the average of 7 and 9. 7 – (0 – 9) equals 16, as usual.
(b) 5 + (4 x 2) = the average of 5 and 8 = 6.5.
(c) 4+6+9 has two values: (4+6)+9 = 5+9 = 7; 4+(6+9) = 4+7.5 = 5.75. Notice that neither answer equals 19/3, the usual (mathematical) average of 4, 6, and 9.
Comment If 0 Wild is chosen along with Average, any 0 that represents + must mean average.

15. Next Prime Number: x*A* means "the next prime number bigger than *A*," where *A* is a rational number less than or equal to 200.

Comment This variation does not rule out using x for multiplication. In the Goal or Solution, the meaning of an x cube will usually be clear from the context since next prime is a unary operation and multiplication is a binary operation. For example, the Goal 4xx6 has only one interpretation: 4 x (x6), which is 4 x 7 = 28.

(a) x7 = 11, the next prime number bigger than 7.

(b) x(9 ÷ 2) = 5, the next prime bigger than 4.5.

(c) x(0 – 3) = 2, the next prime number bigger than –3. (Note: 1 is not prime.)

(d) xx5 = x(x5) = x7 = 11.

(e) x√49 = x7 = 11. x√67 is undefined because √67 is not a rational number.

(f) In the expression 2xx5, the first x means multiplication and the second means next prime number. The value of this expression is 2 x (x5) = 2 x 7 = 14.

(g) In the expression x5x7, the first x means next prime number and the second means multiplication. The value of this expression is either x(5x7) = 37, the next prime number bigger than 35, or (x5)x7 = **7 x 7 = 49.**

(h) There is no limit to the number of consecutive x's in an expression, especially with the Multiple Operations variation also in effect. Thus xxx9 = xx(x9) = x(x11) = x13 = 17.

C. Junior Division Variations (grades 9-10)

The following two variations are in effect for *all* shakes in Junior and Senior Divisions.

- Sideways: A cube representing a non-zero number may be used sideways in the Goal or Solution to equal the reciprocal of that number.

  Examples 1 + 2 + 2 = 1 + 2 + 0.5 = 3.5;

  1 ÷ 3 = 1 ÷ (1/3) = 1 x 3 = 3

Comment See Appendix for ways to indicate a sideways cube in an Equation.

- Upside-down: A cube representing a number may be used upside-down in the Goal or Solution to equal the additive inverse of that number.

  Examples 6 x 2 = 6 x (–2) = –12. However, 62 is *not* legal for 6 – 2 or 60 + (–2).

  Comment See Appendix for ways to indicate an upside-down cube in an Equation.

  Note: The Sideways rule and the Upside-down rule may be used on a single-digit numeral at the same time if both variations have been selected, but only in a Solution.

  Examples $\underset{ud}{\underset{sw}{3}} = -\frac{1}{3}$; $\underset{ud}{\underset{sw}{8}} = -\frac{1}{8}$

Note that 0 or x Wild, Base *m* and Multiple of *k* are expanded from Middle variations.

1. 0 or x Wild: The 0 or x cube may represent any symbol on the cubes, but it must represent the same symbol everywhere it occurs (Goal and Solution). Each Equation-writer must specify in writing the interpretation of the 0 or x cube if it stands for anything other than itself in the Equation. The player selecting this variation specifies whether 0 or x (but not both) is wild for the shake.

   Examples for x Wild (for 0 Wild, see the examples for Elementary and Middle Divisions)

   (a) x – (x ÷ 3) = 4, where both x's stand for 6, is a correct Equation.
   (b) (9 x 3) x 5 = 1, where both x's stand for –, is a correct Equation.
   (c) x – (3 x 2) = 2, where the first x is 7 and the second x is +, is not a correct Equation.
   (d) An x in the Goal and any x in the Solution must represent the same symbol. For example, the Equation (4 * 2) – 2 = 2x7 is incorrect since x stands for * (or ^) on the left side and for multiplication (by default) in the Goal.
   ↑
   x

2. Factorial: There are two occurrences of the factorial operator (!) available to be used in the Solution and/or the Goal as the Equation-writer chooses to use them. All uses of ! in the Equation must be in writing. *However, if Multiple of k is also chosen for the shake, no factorial may be placed in the Goal.*

   Comments

   (a) 5! ("5 factorial") means 5 x 4 x 3 x 2 x 1, which equals 120. *n*! is defined only for whole number values of *n*. 0! is defined as 1.
   (b) In the absence of grouping symbols, ! applies to just the numeral in front of it. See Section **VII-B-6** for examples.

   Examples

(a) For the Goal 4 x 30, a Solution of 5! is *not* correct since it contains only one cube.
(b) If the Goal is 9 ÷ 8, an Equation-writer may interpret it as 9! ÷ 8!, which is 9. However, the Solution may *not* contain an ! since both allotted factorial signs have been placed in the Goal (unless Multiple Operations has been called – see below).
(c) The Equation 5 x 4 ÷ 0! = 20 is correct since 0! equals 1 as defined in mathematics texts.
(d) The Equation (8 – 5)! + 2 = 4! ÷ 3 is correct since the Goal is (4 x 3 x 2) ÷ 3 = 4 x 2 = 8.
(e) 3!! = (3!)! = (3 x 2)! = 6! = 6 x 5 x 4 x 3 x 2 = 720

3. Multiple Operations: Every operation sign in Required or Permitted may be used many times in any Solution. If the Factorial variation is also chosen for the shake, an unlimited number of factorial operators may be used in each Solution. At most two factorials may be used in the Goal.

Comments
(a) After an Impossible challenge, any operation sign in Resources may be used many times in a Solution. After a Now challenge, if the one cube allowed from Resources is an operation sign, it may be used multiple times.
(b) Players may simply write an operation sign multiple times in Solutions without any additional comment since an operation cube is not being used to represent another symbol.

4. Any Color Exponent: Any numeral on a ______cube may be used as an exponent without being accompanied by an * (or ^) cube. The player selecting this variation fills the blank in the previous sentence with one of the colors red, blue, green, or black. The player should announce "Red Exponent" or "Blue Exponent", etc. to indicate the color.

Examples
(a) If the chosen exponent color is red, the Goal 253, where the 3 is red, must mean 25^3 since three-digit numerals are illegal.
(b) If blue is the chosen color, a Goal or a Solution like 5^2, where the 2 is on a blue cube, is legal.
(c) If red is the chosen color, an expression like 523 [2 and 3 red] must be interpreted as either 52^3, or $(5^2)^3$, which is 5^6. It may *not* be interpreted as 5*(2*3) or 5^8, because the 2 by itself is no longer an exponent of the 5.

Comments
(a) If Factorial is also chosen, a ! may be placed behind an exponent. So with red exponent, a Goal of 23 (red 3) may be interpreted as $2^{3!}$ or 2^6.
(b) If a player selects Any Color Exponent when no digits of the selected color were rolled, that player is penalized one point and must pick another variation.

5. Base *m*: Both the Goal and the Solution must be interpreted as base *m* expressions, where the player choosing this variation specifies *m* for the shake as eight, nine, ten, eleven, or twelve. Two-digit numerals are allowed in Solutions. For bases eleven and twelve, * (or ^) may be used for the digit ten; in base twelve, √ may be used for the digit eleven.

If Sideways and Base Eleven (or Twelve) are both chosen, an * (or ^) may be used sideways to represent one-tenth. If the * (or ^) is part of a two-digit numeral, it may not be interpreted as sideways. If an * (or ^) is a one-digit numeral in the Goal, the Equation-writer may interpret the * (or ^) as right-side up or sideways regardless of the way the * (or ^) is physically placed in the Goal. In a Solution, the writer must clearly indicate if an * (or ^) is sideways.

If Upside-down and Base Eleven (or Twelve) are both chosen, an * (or ^) may be used upside-down to represent -10. If the * (or ^) is part of a two-digit numeral, it may not be interpreted as upside-down. If an * (or ^) is a one-digit numeral in the Goal, the Equation-writer may interpret the * (or ^) as right-side up or upside-down regardless of the way the * (or ^) is physically placed in the Goal. In a Solution, the writer must clearly indicate if an * (or ^) is upside-down.

Note: For newer games with ^ on the cubes instead of *, a ^ must *never* be placed sideways or upside-down in the Goal or written sideways or upside-down in the Equation. This is consistent with the principle that ^ inherits all the properties of *, one of which is that * is ambiguous with regard to sideways and upside-down.

*Examples (in each case, * may be replaced by ^.)*
(a) For Base eight, 37 + 5 = 6 * 2 is a correct Equation. Any Solution or Goal containing the digit "8" or "9" is an

illegal expression.

(b) For Base nine, 34 + 5 = 6 * 2 is a correct Equation. Any Solution or Goal containing the digit “9” is an illegal expression.

(c) In Base eight, a Goal like 3 + 8 or 39 should be challenged Impossible. A Goal like 39 is also illegal in Base nine.

Comments

(a) In bases eleven and twelve, * (or ^) may still represent exponentiation; in base twelve, √ may still represent root. If the interpretation of an * (or ^) or √ is not clear from the context of the Solution, the Equation-writer must indicate which meaning is desired so as to eliminate any ambiguity in the Solution or the Goal. (See Appendix.)

(b) If Powers of the Base is chosen with Base Eleven, 1 may mean one, eleven, one-hundred twenty-one, and so on, or one-eleventh, one one-hundred twenty-first, and so on. If Powers of the Base is chosen along with Base Twelve, 1 may mean one, twelve, one-hundred forty-four, and so on, or one-twelfth, one one-hundred-forty-fourth, and so on.

(c) If 0 (or x) wild is chosen along with Base Eleven or Twelve, a wild cube may represent * (or ^) for ten or √ for eleven (or exponentiation or root as long as each wild cube represents the same symbol).

6. Powers of the Base: 1 (one) may represent any integral power of ten. (If 1 is used in a two-digit numeral, it stands for 1.) If Base *m* is also chosen, 1 represents any integral power of *m*.

Examples

(a) For Base ten, 9 + 1 may be interpreted as 9 + 1 (since $10^0 = 1$), 9 + 10, 9 + 100, 9 + 1000, and so on, or as 9 + 0.1 (since $10^{-1} = 0.1$), 9 + 0.01, 9 + 0.001, and so on.

(b) If Base eight is chosen, then 1 may represent one, eight, sixty-four, and so on, or one-eighth, one-sixty-fourth, and so on. For Base nine, 1 represents one, nine, eighty-one, one-ninth, etc.

7. Multiple of *k*: A Solution must not equal the Goal but must differ from the Goal by a non-zero multiple of *k*, where the player choosing this variation specifies *k* for the shake as a whole number from six to twelve, inclusive.

Example If *k* = 6 and the Goal is 5, then a Solution must equal 11, 17, 23, 29, and so on, or –1, –7, –13, –19, and so on. A Solution equal to 5 is incorrect.

Comment Multiple of *k* does not require any special writing of the Goal by an Equation-writer. As always, write the interpretation of the Goal, indicating wild cubes and grouping. You do not have to indicate the multiple of *k* difference.

8. Number of Factors: x*A* means “the number of counting number factors of *A*,” where *A* is a counting number.

Comment Since there is no limit to the size of *A*, it is possible to present an Equation that is uncheckable. For example, with Multiple of *k* =11 and Factorial:

x(8!! + 1) = 5

Any such Equation that cannot be verified (even with a calculator) by opponents and judges as correct or incorrect *will be ruled incorrect*.

Comments

(a) This variation does not rule out using x for multiplication. In the Goal or Solution, the meaning of an x cube will usually be clear from the context since number of factors is a unary operation and multiplication is a binary operation.

(b) If 0 Wild is chosen along with Number of Factors, one 0 may represent number of factors while another 0 may be multiplication since 0 is the symbol x in both cases.

Examples

(a) x(6 x 2) = 6 (since 12 has six factors: 1, 2, 3, 4, 6, 12)

(b) x(4 x 4) = 5 (since the factors of 16 are 1, 2, 4, 8, 16)

(c) x0, x(1 ÷ 2), x(1 – 4) are not defined.

(d) xx12 = x(x12) = x6 = 4

(e) In the expression 3xx7, the first x means multiplication and the second means number of factors. 3xx7 = 3 x (x7) = 3 x 2 = 6.

(f) In the expression x4x2, the first x means number of factors and the second means multiplication. By default, the value of this expression is (x4)x2 = 3 x 2 = 6. To obtain the number of factors of 8, the

Equation-writer must write x(4x2).

The following odd-year variations will be played in 2018-19:

9. AB+: The Goal and/or the Solution may be or may include a three-cube expression of the form AB+, which is interpreted as a repeating decimal. It may be interpreted as .ABABAB... or as .ABBBBB... A player who presents an Equation must specify in writing which interpretation is used in the Goal and Solution. No decimal points may be used in the Equation (except when the decimal point variation is also chosen for the shake). When the form AB+ is used in an Equation, the Equation-writer should indicate in writing which fraction the AB+ is used for (.ABAB or .ABBB).

Examples

(a) 33+ = 0.333333…(.ABAB interpretation, which is 33/99 and simplifies to 1/3) or 0.333333 (.ABBB interpretation, which also simplifies to 1/3).

(b) 90+ = 0.909090…(.ABAB interpretation, which is 90/99 and simplifies to 10/11) or 0.900000 (.ABBB interpretation, which is [90-9]/90, which simplifies to 9/10).

(c) 1-(55+) = (99/99) – (55/99) = 44/99 (which simplifies to 4/9 in either interpretation).

Comments

(a) If Base *m* is also chosen, the decimal places also change to the appropriate base *m* expression, meaning the fractions change to AB/77 for Base 8, AB/88 for Base 9, etc.

(b) In Base *m*, the numerators and denominators in fractions also must be converted to the appropriate base.

10. Add to Goal: On his turn, instead of a regular move, a player may physically add a cube to the Goal. The cube may be placed anywhere in the Goal. However, the limit of six cubes in the Goal, with no numeral containing more than two consecutive digits, still prevails.

Comments

(a) A goal must be set before a challenge is made (i.e., at least one cube must be on the goal line).

(b) The original Goal-setter must put at least one cube on the Goal line. If he states that the Goal is set with no cubes (the goal line is blank), a one-point penalty is assessed.

The following even-year variations will be played again in 2019-20

11. += Average: + shall not represent addition; instead, it shall represent the operation of averaging *two* numbers.

Examples

(a) 7 + 9 = 8, the average of 7 and 9. 7 – (0 – 9) equals 16, as usual.

(b) 5 + (4 x 2) = the average of 5 and 8 = 6.5.

(c) 4+6+9 has two values: (4+6)+9 = 5+9 = 7; 4+(6+9) = 4+7.5 = 5.75. Notice that neither answer equals 19/3, the usual (mathematical) average of 4, 6, and 9.

Comment If 0 Wild is chosen along with Average, any 0 that represents + must mean average.

12. Next Prime Number: x*A* means "the next prime number bigger than *A*," where *A* is a rational number less than or equal to 200.

Comment This variation does not rule out using x for multiplication. In the Goal or Solution, the meaning of an x cube will usually be clear from the context since next prime is a unary operation and multiplication is a binary operation. For example, the Goal 4xx6 has only one interpretation: 4 x (x6), which is 4 x 7 = 28.

(a) x7 = 11, the next prime number bigger than 7.

(b) x(9 ÷ 2) = 5, the next prime bigger than 4.5.

(c) x(0 – 3) = 2, the next prime number bigger than –3. (Note: 1 is not prime.)

(d) xx5 = x(x5) = x7 = 11.

(e) x√49 = x7 = 11; x√67 is undefined because √67 is not a rational number.

(f) In the expression 2xx5, the first x means multiplication and the second means next prime number. The value of this expression is 2 x (x5) = 2 x 7 = 14.

(g) In the expression x5x7, the first x means next prime number and the second means multiplication. The value of this expression is either x(5x7) = 37, the next prime number bigger than 35, or (x5)x7 = 7 x 7 = 49.

(h) There is no limit to the number of consecutive x's in an expression, especially with the Multiple Operations variation also in effect. Thus xxx9 = xx(x9) = x(x11) = x13 = 17

D. Senior Division Variations (grades 11-12)

Players may choose any of the Junior variations (except for the two that are in effect for every shake) plus the following:

13. *Imaginary*: | (sideways minus) shall represent the imaginary number *i* (such that $i^2 = -1$). The | may be placed immediately before or after a numeral without the x sign. When this variation is selected, all roots of a^b, where a is a complex number and b is a rational number are available. Each Equation-writer must write *I* in the Equation (Solution and/or Goal) for the Imaginary unit.

Note: **This variation may be selected even if no – signs (or wild cubes) are available in Resources.**

With this variation, the rules for legal expressions in Section **III-C** are amended to allow expressions like *a* * (*b* ÷ *c*) [or *a* ^ (*b* ÷ *c*)] where *a* is a negative real number, *b* is an integer, and *c* is an even non-zero integer (when *b* ÷ *c* is reduced to lowest terms). Furthermore, in a Goal or Solution, any expression of the form *a* * (*b* ÷ *c*) [or *a* ^ (*b* ÷ *c*)] (where *c* ≠ 0) may equal any one of the complex roots equal to the expression. An Equation-writer using such an expression must indicate in writing which one of the complex roots the expression equals. [See examples **(f) and (g)** and comment **(d)** below.]

If | is multiplied by a number before or after it, as allowed by this variation without a x sign, the implied multiplication takes precedence over any other operations in the expression (in the absence of parentheses). See the examples below.

A player may use an x before or after a |. In that case, the explicit multiplication involving the *i* also takes precedence over other operations.

*Comments (in each case, * may be replaced by ^.)*

(a) "Numeral" means "any expression that names a number, real or otherwise." *i* itself is a numeral, which means that expressions like **||**, **|||**, and so on, are legal and equal i^2, i^3, etc.

(b) An expression like 4***|** is not allowed because the exponent is not a real number. However, **|***4 is permitted**.**

(c) | is ambiguous as regards right-side up and upside down. The default is right-side up. So any **|** in the Goal may be interpreted as *i* or -*i.* However, **|** My not be interpreted as sideways.

(d) If 0 (or x) wild is also chosen, any wild cube may be used as **|** to equal *i* or -*i.* Another wild cube may also be used for – in the same Equation since it stands for the same *symbol* in both cases. If Multiple Operations is also chosen, the wild cube may be used multiple times, but only once as *i.*

(e) With Imaginary, the Goal and the Solution may equal non-real (complex) numbers.

(f) If Multiple Operations is also chosen, **|** may *not* be used multiple times because it represents a numeral and not an operation.

(g) A Goal like 2**|***88 may *not* be interpreted as 2(*i**88) since the variation allows a numeral in front of **| without x but not in front of an expression like |*88 without a x.** Similarly, the expression 2**|***8 in a Solution must be interpreted as (2**|**)*8 and not 2(**|***8) whether the Equation-writer includes the parentheses around "2**|**" or not.

*Examples (in each case, * may be replaced by ^.)*

(a) 2*i* may be represented in a Goal or Solution by either 2**|** or **|**2. ㄣ**|** or **|**ㄣ is ±0.25*i*.
Ɛ**|** or **|** Ɛ is ±3*i*.

(b) 3 + 4*i* may be represented as either 3 + 4**|** or 3 + **|**4.

(c) (3+4) **|** or **|** (3+4) equals 7*i*.

(d) i^6 may be represented as **|** * 6.

(e) 14*i* may be represented as 7**|** 2.

(f) A Goal of 4 * (1 ÷ 2) may equal 2 or –2. A Goal of 16 * ㄣ may equal 2, –2, 2*i*, or –2*i*. Each Equation-writer must eliminate any such ambiguities in his Equation.

(g) Suppose the Goal is 0 – 8 **|**. Then a Solution might be this: (8 x 2) * (3 ÷ 4). The Equation-writer must indicate in a clear and unambiguous manner which root is being used. One way is this:

$$\underbrace{(8 \times 2) * (3 \div 4)}_{(2i)^3}$$

Examples of the default order of operations with | = i

Expression	Default Interpretation	Expression	Default Interpretation
(a) 3+2|	3+(2|)	(b) |2+3	(|2)+3
(c) | |*2	(| |)*2	(d) 5| |2–7	(5| |2) –7
(e) 3+2x|	3+(2x|)	(f) |√4	|(√4)
(g) √9+|	(√9)+|	(h) 5x|–|4	(5x|)–(|4)

(i) The expression √9| contains a clash of defaults. The rule for √ says it must be interpreted as (√9)| but the default for | says it must be √(9|). So the Equation-writer must indicate which interpretation they want to use.

(j) If an expression like 3+2| is in the Goal, a player may interpret it as (3+2)| but must write the parentheses on the right-side of the Equation.

14. <u>Decimal in Goal:</u> Each Equation-writer may determine where decimal points occur in the Goal. Three consecutive digits may be placed in the Goal, but a decimal point must be placed in front of them, between two of them, or after the third digit in the Goal of any Equation.

Examples

(a) A Goal of 20 may be interpreted as 20, 2.0, or 0.2

(b) A Goal of 2 * 3 may be interpreted as 2 * 3, 0.2 * 3, 2 * 0.3, or 0.2 * 0.3. (Each * may be replaced by ^ for newer games.)

(c) 125 in the Goal must have a decimal point inserted to give: .125, 1.25, 12.5, or 125.

Comment A decimal point may be placed in front of only a right-side-up digit. Therefore, no decimal point may be placed in front of a sideways or upside-down cube or in front of *i* (since *i* is not a digit).

15. <u>÷ as log:</u> ·|· (sideways ÷) represents the log operation. Thus, if *a* and *b* are positive real numbers ($b \neq 1$), a ·|· b equals $\log_b a$. A sideways ·|· sign means that log must be used; a normal ÷ sign is ambiguous and can be log or regular division.

Examples

(a) [(6 x 4) + 1] ·|· 5 = $\log_5 25$ = 2.

(b) 3 2|= $\log_2 3$, which is an irrational number.

(c) *a* 1|is undefined for any value of *a*. 0 ·|· 5, (0 –1) ·|· 2, (0 – 8) ·|· (3 – 1), and 4 ·|· (0 – 2) are all undefined.