MICHIGAN LEAGUE OF ACADEMIC GAMES OFFICIAL ADVENTUROUS RULES FOR EQUATIONS 2011-2012

An Explanatory Remark:
➔ indicates a change from the previous year's rules.

ET1 The OFFICIAL RULES FOR BASIC EQUATIONS as printed by the Michigan League of Academic Games and the Tournament version of ADVENTUROUS VARIATIONS OF EQUATIONS (see rules ET4 and ET6) will be used at all levels.

ET2 Two- or three-person games will be played. The Goal-setter for the first shake will be determined by lot (eg., each player rolls a digit cube of the same color, with highest digit going first). On each subsequent shake the Goal-setter is the person immediately to the left of the last Goal-setter.

ET3 A shake will include 24 cubes, six of each color.

ET4 To begin a shake, the Goal-setter has one minute to roll the cubes and study the Resources. At the end of this minute, the Goal-setter has 15 seconds to write and announce to the opponent(s) the name of the variation(s) the Goal-setter has chosen for the shake. These variations will be written on a common sheet of paper. The variation(s) must be selected from the appropriate list in ET6. Then, proceeding to the left, each additional player must write and say the name of the variation(s) she has chosen. She must do so within about 15 seconds after the preceding player has revealed a valid selection or within about 15 seconds after the original minute, whichever is longer. (If a player selects a variation like Base m, the player must also fill in which base is chosen.) If a player selects an illegal variation (e.g., a Senior variation in Junior Division or "base five") or a variation that a previous player has already chosen for that shake (eg., k = 9 when k = 7 has already been chosen or

base eight when base nine has already been called or, in On-Sets, B wild when G wild has already been chosen) or a variation that has no effect because the cube involved is not in the Resources rolled (e.g., 0 wild when there are no 0's, or + = average when no +'s were rolled, etc.),
then that player loses one point and has 15 seconds to write the name of another variation. If, on this second try, that player still does not select an appropriate variation, then he loses another point and may not play that shake (i.e., scores 6 for the shake, which, combined with the two -1's, gives a net score of 4). If this happens in a two- player match, then the other player scores 10 and the next shake is begun (assuming the end-of-round warning has not been given). In two-player matches in Elementary, Middle, and Junior Divisions, the player who is not the Goal-setter must select two variations for the shake.

When a player selects a variation that can’t be used in a game and this selected variation is insulated by the picking of a correct variation, it is ignored and the players play only with the legal variations.

In Senior Division any player may pick two variations. When a player selects two variations, both names must be written within the 15-second time limit. Also two different variations must be selected. Thus a player may not, for example, pick both multiple of 6 and multiple of 8 or, in On-Sets, B required and ∪ required.

ET5 A shake begins as soon as the stall for rolling the cubes has begun or a player has picked up the cubes to roll them. Once the cubes are rolled to form the Resources, no player may alter the face of the cubes nor obstruct the other players' view of any cubes remaining in Resources.

ET6 ADVENTUROUS VARIATIONS

In all divisions a system of even-odd variations is in effect. In this system some variations are used every year while others alternate on an even/odd-year basis. For the 2011-12 school-year, the even-year variations are in effect and are listed for each division. The odd-year variations are also listed.

I. ELEMENTARY VARIATIONS (grade 6 and below)

Note: {counting numbers} = {natural numbers} = {positive integers} = {1, 2, 3, 4, ...}   {whole numbers} = {0, 1, 2, 3, 4, ...}

GENERAL RULE: If * is used for raising to a power, both base and power must be whole numbers. If √ is used for the root operation, the index must be a counting number and the base and total value must be whole numbers.

<u>Sideways cube</u> A cube representing a non-zero number may be used sideways in the Goal or a Solution to equal the reciprocal of the number it represents.

Examples: 1 + 2 + N = 1 + 2 + .5 = 3.5.
1 ÷ ɷ = 1 ÷ (1/3) = 1 x 3 = 3

<u>Upside-down cube</u> In the Goal or a Solution, any numeral may be used upside down to equal the additive inverse of the number represented by that numeral.

Examples: In the Goal or a Solution, 6 x Ƨ = –12. However, 6Ƨ is not legitimate for 6 – 2 or for 60 + (–2).

Note: The **Sideways Rule** and the **Upside-down Rule** may be used on a single digit numeral at the same time in a solution only. Examples: 2 = –1/2 or 5 = –1/5
. sw/usd sw/usd

<u>0 wild</u> The 0 cube may vary and equal any numeral on the cubes, but it must equal the same numeral everywhere it occurs (Goal and Solution). The interpretation of the 0 cube in the Solution is specified in writing by each player who has the burden of proof as part of the Solution. (If 0 stands for 0 in a Solution, this fact need not be specified in writing.) Also a 0 in the Goal and a 0 in the Solution must equal the same number.

Examples: (0 x 6) – 0 =15 where both 0's stand for 3, is allowed but (0 x 6) – 0 = 14 where the first 0 stands for 3 and the second for 4, is NOT allowed; and (0 x 6) – 0 = 12, where the first 0 stands for 2 and the second for 0 is NOT allowed.

Factorial There are two occurrences of the factorial operator (!) available, like parentheses, to be used in the Solution and/or to modify the Goal. All uses of ! in the Solution must be in writing. The player with the burden of proof must explain the uses of ! in the Goal. In the absence of grouping symbols, the unary operation,!, takes priority over the binary operations.

Examples: In the Goal, 4 x 3 may be interpreted as 4 x 3 = 12, (4!) x 3 = 72, 4 x (3!) = 24, (4!) x (3!) = 144, or (4x3)! = 12!. A solution of (4!) x 2 equals a goal of 48. A solution of 5! ÷ (5x2) equals a goal of 12. Use parentheses to avoid making an ambiguous solution.

The following **EVEN-YEAR** variations are in effect for 2011-2012.

Two-digit numerals Two-digit numerals are allowed in Solutions.

Percent —^ means "percent of." That is, A —^ B = A% of B where A and B are numbers. In the Goal or a Solution, A and/or B may be a two-digit numeral.

Examples: In the Goal or a Solution, 6 —^ 8 = 6% of 8 = .06 x 8 = .48. In a Solution (8 - 3) —^ (4 + 2) = 5% of 6 = .05 x 6 = .3. In the Goal or a Solution, 25 —^ 16 = 25% of 16 = .25 x 16 = 4 If the decimal point variation is also in effect, an expression like 1.5 —^ 25 is legitimate in a Solution and equals 1.5% of .25 = .015 x .25 = .00375. Similarly, in a Solution 25 —^ (1.5 x 2) = 25% of 3 = .75.

➔**Decimal point** * or ^ may be used as a decimal point. If so used in the Goal or a Solution, a * or ^ may be combined in a numeral with at most three digits. When used as a decimal, * or ^ take precedence over all other operations.

Note: This variation does not rule out using * or ^ for exponentiation. Therefore, in Solutions players are encouraged to write a decimal point instead of * or ^ when they want to interpret * or ^ as a decimal point. Also one * or ^ may be used as a decimal point and another for exponentiation.

Examples: 2*5 means 2.5 or 2^5. 3*0 means 3.0 or 3^0. 2 * 4 x 2 = 2.4 x 2 or 2^8 or 2^4 x 2; it does <u>not</u> mean 2.8 [that is, it may not be grouped as 2. (4x2)]. In the Goal or in a Solution, *25 means .25 (no exponent interpretation). 12*5 means 12.5 or (in the Goal only unless two-digit numerals has been chosen) 12^5. 1*25 means 1.25 or (in the Goal) 1^{25}. 15*0 means 15.0 or (in the Goal) 15^0. In the Goal or in a Solution, 255* means 255 and *050 means .05. 15*25 = 15^{25} (in the Goal) but has no legitimate interpretation as a decimal. 122*5, 1*225, *1225, and 1225* have no interpretations as decimals or as powers. The expression *37*5 may not be interpreted as .37 1/2 and has no defined interpretation in Elementary Division. Further the "digits" are the symbols 0,1,2,3,4,5,6, 7,8, and 9. A digit turned sideways or upside down is no longer a digit. Therefore, with sideways cube or upside-down cube in effect, you may <u>not</u> use an expression like *ᘖ, ᘖ*, *ᄅ, or ᄅ* and interpret the * as a decimal point.

<u>Number of factors</u> x may be used to indicate the number of counting number factors of a counting number, including the number itself and one. That is, xA = the number of factors of A, where A is a counting number ≤ 200.

Examples: x(6 x 2) = 6 (12, 1, 6, 2, 3, 4)
x(4 x 4) = 5 (16, 1, 2, 8, 4)
x12 = 6 (for use in the Goal or in a Solution if two-digit numerals is also chosen)
x0, x(1 ÷ 2), x(1 –4), and x(5 * 4) are not defined.
xx12 = x(x12) = x6 = 4

<u>Three-operation Solution</u> Any Solution must contain at least three operation symbols. The operation symbols are +, –, x, ÷ ,!, *, and √ .

The following **ODD-YEAR** variations will be in effect for 2012-13.

➔**Multiple operation** Every operation sign in Resources, Required or Permitted may be used many times in a Solution.

Next prime number xA means “the next prime number after A,” where A is a rational number ≤ 200.

Examples: x 7 = 11, the next prime number after 7.
x (9 ÷ 2) = 5, the next prime number after 4.5
x (0 – 3) = 2, the next prime number after –3
2 x x 5 = 2 x 7 = 14. xx5 = x (x5) = x 7 = 11

+ = average + shall not represent addition but instead shall represent the operation of averaging two numbers.

Examples: 7 + 9 = the average of 7 and 9 = 8
5 + (4 x 2) = the average of 5 and 8 = 6.5.

LCM √ may be used to indicate the LCM (least common multiple) of two counting numbers.

Examples: 6 √ 8 = 24; (2 x 3) √ (5 + 4) = 6 √ 9 = 18;
2 √ 4 = 4 (or 2, the square root of 4); 0 √ 5 is undefined.

GCF * may be used to indicate GCF (greatest common factor).

Examples: 8*6 = 2 (or 8 to the power of 6);
(4 x 2)*(6 + 3) = 1 (or 8 to the power of 9).

Note: GCF(A*B) (“the greatest common factor of A and B”) is defined if A and B are counting numbers or if A is a counting number and B = 0 or if B is a counting number and A = 0.

➔**Remainder** A ⸭ B (⸭ is a sideways ÷) equals the remainder when A is divided by B. A and B are positive integers, and A is less than or equal to 1000.

Examples: 15·|·2 = 1, since 15 divided by 2 gives a quotient of 7 with a remainder of 1.
897·|·115 = 92
45·|·70 = 45 [In general A·|·B = A when A < B]
87·|·87 = 0
54·|·10 = 4 [In general, A·|·10 = the last digit of A].

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II. MIDDLE VARIATIONS (grades 7-8)

Sideways cube A cube representing a non-zero number may be used sideways in the Goal or a Solution to equal the reciprocal of the number it represents.

Examples: 1 + 2 + ᴎ = 1 + 2 + .5 = 3.5.
1 ÷ ɯ = 1 ÷ (1/3) = 1 x 3 = 3

Upside-down cube In the Goal or a Solution, any numeral may be used upside down to equal the additive inverse of the number represented by that numeral.

Examples: In the Goal or a Solution, 6 x ᘔ = –12., 8 * 1= 8^{-1} = 1/8. However, 6ᘔ is not legitimate for 6 - 2 or for 60 + (-2).

Note: The **Sideways Rule** and the **Upside-down Rule** may be used on a single digit numeral at the same time in the solution only.
Examples: $2_{sw/ud}$ = –1/2 or $5_{sw/ud}$ = –1/5

0 wild The 0 cube may vary and equal any numeral on the cubes, but it must equal the same numeral everywhere it occurs (Goal and Solution). The interpretation of the 0 cube in the Solution is specified in writing by each player who has the burden of proof as part of the Solution. (If 0 stands for 0 in a Solution, this fact need not be specified in writing.)

Examples: (0 x 6) – 0 = 15 where both 0's stand for 3, is allowed but (0 x 6) – 0 = 14 where the first 0 stands for 3 and the second for 4, is NOT allowed; (0 x 6) – 0 = 12, where the first 0 stands for 2 and the second for 0 is NOT allowed. Also a 0 in the Goal and a 0 in the Solution must equal the same number.

Base m Both the Goal and the Solution must be interpreted as base m expressions, where the player choosing this variation specifies m for the shake as a whole number from eight to ten, inclusive. Two-digit numerals are allowed in Solutions.

Examples:For m = 8, $(37 + 5)_{\text{eight}} = (6 * 2)_{\text{eight}}$ is a correct Solution.

Note: For m = 8, any Solution with an “8” or “9” in it is automatically incorrect. Similarly, any Goal containing an “8” or “9” is impossible. Likewise, when m=9, a Goal with a “9” is impossible and a Solution with a “9” is automatically incorrect.

Factorial There are two occurrences of the factorial operator (!) available, like parentheses, to be used in the Solution and/or to modify the Goal. All uses of ! in the Solution must be in writing. The player with the burden of proof must explain the uses of ! in the Goal. In the absence of grouping symbols, the unary operation,!, takes priority over binary operations.

Examples: In the Goal, 4 x 3 may be interpreted as 4 x 3 = 12, (4!) x 3 = 72, 4 x (3!) = 24, (4!) x (3!) = 144, or (4x3)! = 12!

A solution of (4!) x 2 equals a goal of 48. A solution of 5! ÷ (5x2) equals a goal of 12. Use parentheses to avoid making an ambiguous solution.

The following EVEN-YEAR variations are in effect for 2011-2012.

Percent —^ means “percent of.” That is, A —^ B = A% of B where A and B are numbers. In the Goal or a Solution, A and/or B may be a two-digit numeral.

Examples: In the Goal or a Solution, 6 —^ 8 = 6% of 8 = .06 x 8 = .48 . In a Solution (8 - 3) —^ (4 + 2) = 5% of 6 = .05 x 6 = .3 In the Goal or a Solution, 25 —^ 16 = 25% of 16 = .25 x 16 = 4 . If the decimal point variation is also in effect, an expression like 1.5 –^.25 is legitimate in a Solution and equals 1.5% of .25 = .015 x .25 = .00375. Similarly, in a Solution 25 –^ (1.5 x 2) = 25% of 3 = .75.

Note: If base m and percent are both chosen, the meaning of percent ("per 100") changes according to the base. For base eight, "percent" means "per sixty-four." For base nine, "percent" means "per eighty-one."
Example: If m = 8, then 60 –^ 11 = $(60_{eight} \div 100_{eight})$ x 11_{eight} = $(48_{ten} \div 64_{ten})$ x 9_{ten} = $(3_{ten} \div 4_{ten})$ x 9_{ten} = $27_{ten} \div 4_{ten}$ = $(6\ 3/4)_{ten}$ = $(6\ 6/8)_{ten}$ = 6.6_{eight}.

➔**<u>Decimal point</u>** * or ^ may be used as a decimal point. If so used in the Goal or a Solution, a * or ^ may be combined in a numeral with at most three digits. When used as a decimal, * or ^ take precedence over all other operations.

Note: This variation does not rule out using * or ^ for exponentiation. Therefore, in Solutions players are encouraged to write a decimal point instead of * or ^ when they want to interpret * or ^ as a decimal point. Also one * or ^ may be used as a decimal point and another for exponentiation.

Examples: 2*5 means 2.5 or 2^5 . 3*0 means 3.0 or 3^{0}. 2 *4 x 2 = 2.4 x 2 or 2^8 or 2^4 x 2; it does <u>not</u> mean 2.8 [that is, it may not be grouped as 2. (4x2)]. In the Goal or in a Solution, *25 means .25(no exponent interpretation). 12*5 means 12.5 or (in the Goal only unless base m is also chosen) 12^5. 1*25 means 1.25 or (in the Goal) 1^{25}. 15*0 means 15.0 or (in the Goal) 15^0. In the Goal or a Solution, 255* means 255. , *050 = .05 and 15*25 = 15^{25} (in the Goal) but has no legitimate interpretation as a decimal. 122*5, 1*225, *1225, and 1225* have no interpretations as decimals or as powers. The expression *37*5 means only $.37^5$ and may not be interpreted as .371/2. Further, the "digits" are the symbols 0,1,2,3,4,5,6,7,8, and 9. A digit turned sideways or upside down is no longer a digit. Therefore, with sideways cube or upside-down cube in effect, you may <u>not</u> use an expression like *∾, ∾*, *Ƨ, or Ƨ* and interpret the * as a decimal point.

Number of factors x may be used to indicate the number of counting number factors of a counting number, including the number itself and one. That is, xA = the number of factors of A, where A is a counting number ≤ 200.

Examples: x(6 x 2) = 6 (12, 1, 6, 2, 3, 4)
x(4 x 4) = 5 (16, 1, 2, 8, 4)
x12 = 6 (in the Goal or in a Solution if base m is chosen)
x0, x(1 ÷ 2), x(1 - 4), and x(5 * 4) are not defined.

AB+ The Goal and/or the Solution may be or may include a three-cube expression of the form AB+. AB+ is interpreted as a repeating decimal. It may be interpreted as .ABABAB... or as .ABBBBB... A player who presents a Solution is correct if the Solution satisfies either interpretation of the Goal. No decimal points may be used in the Solution (except when the decimal point variation is also chosen for the shake). When the form AB+ is used in a solution the solution writer should indicate in writing which fraction the AB+ is used for.

Any Color Exponent The player who picks this variation names the color. The numerals on that color cube may be used as an exponent without an * cube.

The following **ODD-YEAR** variations will be in effect for 2012-13.

Multiple operation Every operation sign in Resources, Required or Permitted may be used many times in a Solution.

Next prime number xA means "the next prime number after A," where A is a rational number ≤ 200.

Examples: x7 = 11, the next prime number after 7.
x(9 ÷ 2) = 5, the next prime number after 4.5.
x(0 - 3) = 2, the next prime number after -3.
2xx5 = 2 x 7 = 14. xx5 = x(x5) = x7 = 11.

Powers of the base 1 (one) may vary and stand for any integral power of ten. (If 1 is used in a two-digit numeral, it stands for 1.) If base m is also chosen, 1 represents any integral power of m.

Examples: For base ten, 9 + 1 may be interpreted as 9 + 1 (since $10^0 = 1$), 9 + 10, 9 + 100, 9 + 1000, etc., or as 9 + .1 (since $10^{-1} = .1$), 9 + .01, 9 + .001, etc.
If base 8 is chosen, then 1 may represent one, eight, sixty-four, etc., or one-eighth, one-sixty-fourth, etc. For base 9, 1 represents nine, eighty-one, one-ninth, etc.

<u>+ = average</u> + shall not represent addition but instead shall represent the operation of averaging two numbers.

Examples: 7 + 9 = the average of 7 and 9 = 8.
5 + (4 x 2) = the average of 5 and 8 = 6.5.

<u>Multiple of k</u> A Solution must not equal the Goal but must differ from the Goal by a non-zero multiple of k, where the player choosing this variation specifies k for the shake as a whole number from six to twelve, inclusive. The Goal must not be greater than 1000 or less than -1000.

Examples: If k = 6 and the Goal is 5, then a Solution must equal 11, 17, 23, 29, etc., or –1, –7, –13, –19, etc

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III. JUNIOR VARIATIONS (grades 9-10)

<u>Sideways cube</u> A cube representing a non-zero number may be used sideways in the Goal or a Solution to equal the reciprocal of the number it represents.

Examples: 1 + 2 + ᴎ = 1 + 2 + .5 = 3.5.
1 ÷ ო = 1 ÷ (1/3) = 1 x 3 = 3

<u>Upside-down cube</u> In the Goal or a Solution, any numeral may be used upside down to equal the additive inverse of the number represented by that numeral.

Examples: In the Goal or a Solution, 6 x Ƶ = –12.
8 * 1= 8^{-1} = 1/8. However, 6Ƶ is not legitimate for 6 - 2 or for 60 + (-2).

Note: The **Sideways Rule** and the **Upside-down Rule** may be used on a single digit numeral at the same time in the solution only.

Examples: $2_{sw/ud} = -1/2$ or $5_{sw/ud} = -1/5$

Powers of the base 1 (one) may vary and stand for any integral power of ten. (If l is used in a two-digit numeral, it stands for 1.) If base m is also chosen, 1 represents any integral power of m.

Examples: For base ten, 9 + 1 may be interpreted as 9 + 1 (since $10^0 = 1$), 9 + 10, 9 + 100, 9 + 1000, etc., or as 9 + .1 (since $10^{-1} = .1$), 9 + .01, 9 + .001, etc.
If base eight is chosen, then 1 may represent one, eight, sixty-four, etc., or one-eighth, one-sixty-fourth, etc.
For base nine, 1 represents nine, eighty-one, one-ninth, etc.

0 wild The 0 cube may vary and equal any numeral on the cubes, but it must equal the same numeral everywhere it occurs (Goal and Solution). The interpretation of the 0 cube in the Solution is specified in writing by each player who has the burden of proof as part of the Solution. (If 0 stands for 0 in a Solution, this fact need not be specified in writing.)

Examples: (0 x 6) – 0 = 15 where both 0's stand for 3, is allowed but (0 x 6) – 0 = 14 where the first 0 stands for 3 and the second for 4, is NOT allowed; (0 x 6) – 0 = 12, where the first 0 stands for 2 and the second for 0 is NOT allowed. Also a 0 in the Goal and a 0 in the Solution must equal the same number.

AB+ The Goal and/or the Solution may be or may include a three-cube expression of the form AB+. AB+ is interpreted as a repeating decimal. It may be interpreted as .ABABAB... or as .ABBBB... A player who presents a Solution is correct if the Solution satisfies either interpretation of the Goal. No decimal points may be used in the Solution (other than the implied decimal point in an AB+ expression). When the form AB+ is used in a solution the solution writer should indicate in writing which fraction the AB+ is used for.

Multiple of k A Solution must not equal the Goal but must differ from the Goal by a non-zero multiple of k, where the player choosing this variation specifies k for that shake as a whole number from six to twelve, inclusive.

Example: If k = 6 and the Goal is 5, then a Solution must equal 11, 17, 23, 29, etc., or –1, –7, –13, –19, etc.

Base m Both the Goal and the Solution must be interpreted as base-m expressions, where the player choosing this variation specifies m for the shake as a whole number from eight to twelve, inclusive. Two-digit numerals are allowed in Solutions. For bases eleven and twelve, * may be used for the digit ten (or for exponentiation); in base twelve, √ may be used for the digit eleven (or for root).

Examples:For m = 8, $(37 + 5)_{eight} = (6 * 2)_{eight}$ is a correct Solution.

Note, however, that for m = 8, any Solution with an "8" or "9" in it is automatically incorrect. Similarly, any Goal containing an "8" or "9" is impossible. Likewise, when m=9, a Goal with a "9" is impossible and a Solution with a "9" is automatically incorrect.

Note: If sideways cube and base eleven (or twelve) are both chosen, a * may be used sideways to represent one-tenth. If the * is part of a two-digit numeral, it may not be interpreted as sideways. If a * is a one-digit numeral in the Goal, the Solution-writer may interpret the * as right-side up or sideways regardless of the way the * is physically placed in the Goal. In a Solution the Solution-writer should indicate in a clear manner whether the * is sideways.

➔Note: The ^ symbol is ambiguous when Base 11 or 12 is called. Although it can only be physically pointed up in a Goal, it can also be interpreted as upside down or sideways if those variations are in effect.

<u>Factorial</u> There are two occurrences of the factorial operator (!) available, like parentheses, to be used in the Solution . All uses of ! in the Solution must be in writing. In the absence of grouping symbols, the unary operation,!, takes priority over the binary operations.

> Examples: A solution of (4!) x 2 equals a goal of 48. The solution (5!) ÷ (5x2) equals a goal of 12. Use parentheses to avoid making an ambiguous solution.

The following <u>EVEN-YEAR</u> variations are in effect for 2011-2012.

<u>Any Color Exponent</u> The player who picks this variation names the color. The numerals on that color cube may be used as an exponent without a * cube.

<u>Number of factors</u> x may be used to indicate the number of counting number factors of a counting number, including the number itself and one. That is, xA = the number of factors of A, where A is a counting number.

> Examples: x(6 x 2) = 6 (12, 1, 6, 2, 3, 4)
> x12 = 6 (in a Goal or in a Solution if base m is chosen).
> x0, x(1 ÷ 2), and x(l – 4) are not defined.

<u>Add to Goal</u> On his turn, instead of a regular move, a player may add a cube to the Goal. The cube may be placed anywhere in the Goal. However, the limit of six cubes in the Goal, with no numeral containing more than two consecutive digits, still prevails. To use this variation; A) A goal must be set before a challenge is made. At least one cube must be on the goal line. B) If a player states that a goal is set before actually setting a goal (the goal line is blank) a –1 point is assessed.

The following <u>ODD-YEAR</u> variations will be in effect for 2012-13.

<u>Multiple operation</u> Every sign in Resources, Required or Permitted may be used many times in a Solution.

Next prime number xA means "the next prime number after A," where A is a real number ≤ 200, and if the real number is raised to an exponent, that exponent must be a rational number.

Examples: x7 = 11, the next prime number after 7.
x(9 ÷ 2) = 5, the next prime number after 4.5.
x(0 – 3) = 2, the next prime number after –3.
2xx5 = 2 x 7 = 14. xx5 = x(x5) = x7 = 11.
x((√2)*3)= x(√8)= the next prime number after √8=3.

+ = average + shall not represent addition but instead shall represent the operation of averaging two numbers.

Examples: 7 + 9 = the average of 7 and 9 = 8.
5 + (4 x 2) = the average of 5 and 8 = 6.5.

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IV. SENIOR VARIATIONS (grades 11-12)

Note – In the Senior division, the Sideways and Upside Down Cube variations are automatically in effect for all shakes.

Senior players may select any of the other Junior variations for this year plus the following:

√ = i shall not represent the root operation but instead may represent the imaginary number i (such that $i^2 = -1$). The √ may be placed immediately before or after a numeral without the x sign.

Examples: 2i may be represented in a Goal or Solution by 2 √ or √ 2.
3 + 4i may be represented as 3 + (4 √) or 3 + (√ 4).
(3 + 4) √ or √ (3 + 4) equals 7i.
i^6 may be represented as √ * 6.
i^2 may be represented as √ √ .
14i may be represented as 7 √ 2.

Notes:
a)"Numeral" means "any expression that names a number.

b) When this variation is in force, rule ETI2 is amended to allow expressions like a * (b ÷ c) where a is negative and c is an even non-zero integer (when b ÷ c is reduced to lowest terms). Furthermore, in the Goal or Solution, any expression of the form a * (b ÷ c) (where c ≠ O) may equal any one of the complex roots equal to the expression. A player using such an expression in a Solution must indicate in writing which one of the complex roots the expression equals. (See ET 18.)

Examples. A Goal of 4 * (1 ÷ 2) may equal 2 or –2 With sideways cube a Goal of 16 *  may equal 2, –2, 2 √, or –2 √. Suppose the Goal is 0 – 8 √. Then a Solution might be this: (8 x 2) * (3 ÷ 4) . The Solution-writer must indicate in a clear and unambiguous manner which root is being used. One way is this:

(8 x 2) * (3 ÷ 4)

(2i) [3]

c) If x wild is also picked for the shake, any x cube used as √ must represent i. (Exception: Base twelve)

d) The variation says √ <u>may</u> represent the imaginary number i, instead of <u>must</u> represent, only because base twelve may be chosen. In this case √ may also equal the digit eleven. Without base twelve √ can represent only i.

<u>x wild</u> The x cube may vary and represent any symbol (numeral or operation) on the cubes, but it must stand for the same symbol everywhere it occurs (Goal and Solution). The interpretation of the x cube is specified in writing by each player who has the burden of proof as part of the Solution. (If x stands for x in a Solution, this fact need not be specified in writing.)

Examples: x – (x ÷ 3) = 4, where both x's stand for 6, is allowed; (9 x 3) x 5 = 1, where both x's stand for – is allowed; but x – (3 x 2) = 2, where the first x stands for 7 and the second x for +, is not allowed. An x in the Goal and an x in the Solution must represent the same symbol.

Note: If one player selects x wild, any x used as an operation may be used arbitrarily many times in a Solution.

Decimal in Goal Each Solution-writer may determine where decimal points occur in the Goal. A Solution is correct if it satisfies at least one such interpretation of the Goal.

Examples: A Goal of 20 may be interpreted as 20, 2.0, or .2. A Goal of 2 * 3 may be 2 * 3, .2 * 3, 2 * .3, or .2 * .3.

÷ as log ÷ may represent the log operation or the division operation. Thus a ÷ b may be interpreted as the quotient of a divided by b (provided b ≠ 0) or, if a and b are positive real numbers (b ≠ 1), as $\log_b a$. If the division sign is turned sideways on the goal, then it must be interpreted as log.

Examples: a ÷ 1 = a under the division interpretation but has no interpretation as a logarithm.
(a ÷ b) ÷ c may be interpreted as $\log_c(a \div b)$ if c and a ÷ b are positive (c ≠ 1); as $(\log_b a) \div c$ if a and b are positive (b ≠1) and
c ≠ 0; or as $\log_c(\log_b a)$ if a, b, c, and $\log_b a$ are positive, c ≠ 1, and b ≠ 1. ((6 x 4) + 1) ÷ 5 = $\log_5 25$ = 2 or ((6 x 4) + 1) ÷ 5 = 5.
(0 - 8) ÷ (3 - 1) = –4 but has no defined interpretation as a log. Similarly 4 ÷ (0 – 2) = –2 but has no log interpretation.
0 ÷ 5 = 0 but has no interpretation as a log. 5 ÷ 0 is undefined under either interpretation.
(0 – 1) ÷ 1 = –1 but has no interpretation as a log.
3 ÷ 2 = 1.5 under the division interpretation; under the log interpretation $\log_2 3$ equals an irrational number.

* *

SPECIAL NOTE: Negative ones are covered under the following rules. These are for all Divisions. Time limits(ET7); Variation writing(ET4); Challenging yourself(ET14); Not specifying a challenge(ET14); and Improper behavior(ET25). Junior and Senior Division has these additional two; Bonusing in the lead(ET11) and Add to the Goal (ET7).

ET7 Time limits will be imposed. Use a one-minute timer. For the time limits for rolling the cubes and selecting variations, see ET4.

For the time limits when a forceout declaration is challenged, see ET22. The rest of the time limits are as follows.

Setting the Goal or declaring “No Goal”......................2 minutes
First turn of the player immediately to the left of the
Goal setter...2 minutes
All other regular turns (including any Bonus moves).... 1 minute
Deciding whom to join on a challenge or deciding whether
or not to accept a “No Goal” declaration.......................1 minute
Writing a Solution after a challenge or after a forceout
declaration that is not challenged.................................2 minutes
(Note: If there is a Third Party, these two minutes begin after
the Third Party has taken a side.)
Deciding whether to accept an opponent's Solution.......2 minutes

NOTE: Except for emergencies, there are no time-outs.

Usually a player sets the Goal, moves, takes a side, etc., before his time expires. Some sand must be allowed to run out before the timer can be set to one minute for the next player. However in practice players usually have more than one or more than two minutes to complete what they must do. Player's stalling an opponent may either flip or not flip the timer, as the case may be, so as to give the opponent the lesser amount of time. If, say, 15 seconds are left from the previous time limit, let this sand run out, then flip the timer to begin the next player's stall. But if, say, 45 seconds remain from the previous player, flip the timer, allow the sand to run out, then flip again to begin the next player's time limit. A player being timed must be warned approximately ten seconds before his time expires. (If none of the players notices the time expire, the player being timed must move within 10 seconds after someone does notice that the time has expired.) If despite the warning his time runs out before he finishes what he must do, he loses a point and has one more minute to complete the task. If he is not finished at the end of this additional minute, he loses his turn.
Note: When calling a judge you cannot keep checking a solution if the timer is put down. If the timer is put down you must turn over the paper and wait for the judge. If the timer is still running you may continue to check the solution.

➔**ET8** The Goal may contain between one and six cubes and may not contain numerals with more than two consecutive digits. Furthermore, if a Goal is set which has no defined interpretation in the game of Equations for that shake, that Goal is a P-Flub and can and should be challenged as impossible. When finished setting the Goal, the Goal-setter should say “Goal” or “Goal Set.” If he does not, then an opponent may ask if he is finished. If he indicates that he is, the shake may proceed. Or, if the Goal-setter's time runs out and he has not said “Goal,” he may revise the Goal only if he takes a one-point penalty (see ET7).

Examples: 43 x 82 is an allowable Goal, but 122 is a P-Flub (unless the any color exponent variation is in effect and the second 2 is the color called, in which case the Goal must be interpreted as 12^2). If upside-down cube is not in force but the Goal-setter puts a cube upside-down in the Goal, then that Goal is a P-Flub. Similarly, if AB+ is not in force, a Goal like 23+ is a P-Flub. If add to Goal is being played and the Goal is set as 23+, the Goal is not impossible since one or two Resource cubes could still be added to it. However, it is illegal procedure since the Goal on the mat should at all times be a legal Equations expression.

ET9 A move which violates a procedural variation or custom is labeled illegal procedure. Examples are moving out of turn or moving two cubes without calling “Bonus” before the first cube touches the mat in Forbidden. If illegal procedure is charged, the player charging illegal procedure must specify (within 15 seconds) what the illegal procedure is. If it is illegal procedure, then the Mover must return the illegal cube(s) to Resources and, if necessary, make another move. When an illegal procedure is proven against a “bonus move”, the first cube moved remains where played and the second cube moved must be returned to resources. The mover’s turn is over. After the illegal procedure is identified, the Mover must be given at least 10 seconds to make this correction, unless the original move was made after the ten-second warning, in which case the Stalling Rule (ET7) is enforced. There is no direct penalty except that the Mover's time limit may expire before he legally completes the turn (see ET7). An illegal procedure is insulated by a legal action by another player so that, if the illegal procedure is not corrected before another player takes a legitimate action, it stands as played. There are three legal actions another player may take: (a) make a legal move; (b) challenge; or (c) call forceout. (Actions (a) and (c) are taken by the player to the left of the previous Mover but if there are three parties in the shake, either of the other two may challenge.) If illegal procedure is charged but the move is not illegal procedure, the cube stands as played. There is no penalty for erroneously charging illegal procedure. (However, see ET25 if a player does so frequently.)

ETl0 No books, tables, calculators, or prepared notes may be used. Only pencils or pens and blank paper may be used. (The host school or the League provides a copy of the Adventurous Variations for each player.)

ET11 The player who is ahead in the match may not make Bonus moves. If two or more players are tied for the lead, they may make Bonus moves. In the Junior and Senior Divisions, if the player in the lead makes a Bonus move and this illegal procedure is charged before being insulated, the Mover loses one point and must return the last cube moved to Resources.

ETl2 The following expressions are undefined (where ‘a’ is any number).

$0 \surd a$, $a \div 0$, $0 * a$ where $a \leq 0$

$a \surd b$ where a is an even integer and b is negative

$(a \div b) \surd c$ where c is negative and, when $a \div b$ is reduced to lowest terms, a is an even integer and b is an odd integer

$a * (b \div c)$ where a is negative and, when $b \div c$ is reduced to lowest terms, b is an odd integer and c is an even integer

Every expression that contains an undefined expression is also undefined. A player who uses an undefined expression does not sustain his burden of proof. A player who sets a Goal with no defined interpretation causes a P Flub.

Examples: Middle, Junior and Senior Divisions only:
Is $(-8)^{4/6}$ defined? First reduce the fractional exponent to lowest terms:

$(-8)^{4/6} = (-8)^{2/3}$

Now apply ET12. $(-8)^{2/3}$ is of the form a * (b ÷ c) where a is negative. Since b is even and c is odd, $(–8)^{2/3}$ is defined. (ET13) tells how to evaluate it.

Is $(–4)^{2/4}$ defined? $(–4)^{2/4} = (–4)^{1/2}$, which is of the form a * (b ÷ c) with a negative, b odd, and c even. Therefore, $(–4)^{2/4}$ is undefined.

Note: The following reasoning is not allowed.
$(–4)^{2/4} = {}^{4}\sqrt{(–4)^2} = {}^{4}\sqrt{16} = 2$ (exponent not reduced)

${}^{3/6}\sqrt{–9} = {}^{1/2}\sqrt{–9}$, (reduce the index to lowest terms) which is of the form (a ÷ b) √ c, with c negative. However, a is odd and b is even. So ${}^{3/6}\sqrt{–9}$ is defined. (See ET13 for its value.)

${}^{8/2}\sqrt{–5}$ is not defined because ${}^{8/2}\sqrt{–5} = {}^{4}\sqrt{–5}$, which is of the form a √ b where a is even and b is negative.

Note: the following steps are incorrect.
${}^{8/2}\sqrt{–5} = {}^{8}\sqrt{(–5)^2} = {}^{8}\sqrt{25}$, which is defined.

ET13 For the EQUATIONS game, even-indexed radical expressions indicate only non-negative roots.

Examples: ${}^{2}\sqrt{9} = 3$ (not –3) ; ${}^{4}\sqrt{16} = 2$ (not –2)

In all cases not covered by ET12, (p ÷ q) √ x where p and q are non-zero integers and p/q is in lowest terms, shall be defined as follows: ${}^{p/q}\sqrt{x} = x^{q/p} = {}^{p}\sqrt{x^{q}} = ({}^{p}\sqrt{x})^{q}$

Examples: Middle, Junior and Senior Divisions only:
$(–8)^{4/6} = (–8)^{2/3} = {}^{3}\sqrt{(–8)^2} = {}^{3}\sqrt{64} = 4$ [first reduce the exponent] or
$(–8)^{4/6} = (-8)^{2/3} = ({}^{3}\sqrt{(–8)})^{2} = (–2)^{2} = 4$ [first reduce the index]
${}^{3/6}\sqrt{–9} = {}^{1/2}\sqrt{–9} = ({}^{1}\sqrt{–9})^{2} = (–9)^{2} = 81$

Incorrect: $(–4)^{2/2} = {}^{2}\sqrt{(–4)^2} = {}^{2}\sqrt{16} = 4$

$(-8)^{2/6} = (-8)^{1/3} = 3\ \surd -8 = -2$ [Exponent reduced first]
Incorrect: $(-8)^{2/6} = {}^{6} \quad (-8)^{2} = {}^{6} \quad 64 = 2$ [Exponent not reduced]

NOTE: If a root or radical sign is used without an index it is understood to be the square root or 2nd root of this number when used in a solution or goal. The number two may be used as an index. All other indexes must be in writing or in the goal. It is strongly suggested that all indexes be written in the Vee of the root sign.

ETl4 A challenge block or flub ball will be placed equidistant from all players. To challenge, a player must <u>pick up</u> the block or flub ball. If she has not picked up the block or ball, she has not challenged. The Challenger must say immediately (within about 15 seconds) why she has picked up the challenge block or flub ball. The challenge may not be retracted. The Stalling Rule, ET7, applies to this situation. If the player who picked up the block does not specify a legitimate challenge within 15 seconds, a one point penalty is enforced. The player has a minute to specify a valid challenge. If no valid challenge is stated within that minute, the player loses another point and the shake continues. The only exception is the case where the player who picked up the block is also the last mover so that the player is in effect challenging herself. In this case the player loses one point, the “challenge” is set aside and the shake continues. Players should not pick up the challenge block to call forceout, illegal procedure, goal set, bonus, or for any reason other than challenging. Picking up a challenge block or flub ball for calling a Forceout, Illegal Procedure, Goal Set, Bonus, or for any other reason other than challenging is an illegal procedure and is not considered a "challenge". The player who made the illegal procedure should have it explained that the flub ball or challenge block should not be picked up. He should then repeat his original statement or move and the game continues. Touching the challenge block has no significance. However, players may not keep a hand or finger on or near the block throughout the shake. (See ET25.)
Players may say “Flub Adjustment” or something similar such as “Flub Ball Adjustment” and then move the Flub Ball to a spot that is an equal distance from all players. This is not a challenge because there was no intent to challenge.

ETI5 In responding to or proving a challenge, once a player presents his Solution to his opponents, he may make no further corrections or additions, even if his time has not expired. If a paper is presented with more than one Solution, the writer of the Solution has the right to indicate which Solution is being submitted.

ETI6 "If an expression offered as a Solution is ambiguous, then the burden of proof is sustained if and only if every interpretation of the expression satisfies at least one interpretation of the Goal." (Green EQUATIONS Rule Book) There is no hierarchy of operations in Adventurous Equations. Therefore it is the duty of the player writing a Solution to use symbols of grouping to specify the desired order of operations. ORALLY STATING THE ORDER IN WHICH OPERATIONS ARE TO BE PERFORMED IS <u>NOT</u> SUFFICIENT. Necessary symbols of grouping must be written as part of the Solution before it is presented to opponent(s) for acceptance or rejection. If an opponent believes there is an interpretation of the ambiguous Solution which does not equal any interpretation of the Goal, then that opponent, during the two minutes allotted for checking the Solution, should copy the Solution on his own paper and add symbols of grouping where he thinks they will create a wrong interpretation. If this revised Solution does not equal the Goal, the original Solution-writer does not sustain the burden of proof. In a three player match, during the two minutes allotted for checking that Solution, either opponent may use this procedure to prove that the ambiguous Solution has an incorrect interpretation. However, each opponent has only one opportunity to show, in writing, an incorrect interpretation. (Of course, either opponent may prove the Solution wrong on other counts--- did not use a cube in Required; used a cube that is not available in Required, Permitted, or Resources; violated a variation, etc.)

Examples: Suppose the Goal is 20. A Solution of 8 x 2 + 4 is incorrect because it may be interpreted as 8 x (2 + 4) = 48. The player writing the Solution should have grouped: (8 x 2) + 4 ,[8x2]+4,or {8x2}+4.

Suppose the Goal is 24. A Solution of 4 x 2 x 3 is correct because either interpretation, (4 x 2) x 3 or 4 x (2 x 3), equals 24. Similarly 8 x 3 + 0 is correct. However, 8 x 0 + 3 is not since it could be interpreted as (8 x 0) + 3 = 3. Also (5 x 4) * 1 + 4 is not correct and should have been written as [(5 x 4) * 1] + 4 or (5 x 4 * 1) + 4 or [5 x (4 * 1)] + 4.

Note: In the case where two unary operations are used, parentheses should be used in a solution to avoid ambiguity. This is true with binary operations as well.

ETI7 "When a Challenger has alleged an A-claim violation or a C-claim violation that stems from a previous A-claim violation, he also has the burden of proving that there was an alternative move that

(a) did not allow a Solution to be built with at most one more cube from the Resources, and

(b) did not violate the P-claim." (EQUATIONS Manual, 1969, pp. 12-3) ."

An A- or CA-Challenger (and the Third Party if siding with the Challenger) must present in writing, along with a Solution, an alternate move ("avoid move"). The Solution and the avoid move must both be presented by the end of the two minutes allowed for writing a Solution. If a correct avoid move is not written with a Solution, the Burden of Proof is not sustained.

Examples: The avoid move might be abbreviated 6 -> F ("6 to Forbidden"), x to Req ("x to Required"), F + ("forbid a +"), R 7 ("require a 7"), etc.

ET18 It is the Solution-writer's burden to make sure the Solution presented is unambiguous. This means that symbols of grouping must be used where necessary. (See ETI6.) It also means that, when variations like 0 wild or AB+ are in force, care must be taken to indicate in writing in a clear and unambiguous manner what meaning each symbol has in the Solution. For each variation there is a "default" interpretation that will take effect if the Solution-writer does not indicate the meaning. A recommended but not exhaustive list of examples of written designations is included below to encourage uniformity. For a more complete list of examples, see Appendix A.

Examples: For 0-wild a player uses 0 to mean 2. So she writes 2 x (8 + 7) as the Solution and indicates that 0 stands for 2. Several ways to do this are to write

2 x (8 + 7) or   2 x (8 + 7).
↓   ↑
0   0

(It is acceptable, but not encouraged, to write “0” in the Solution, with the “2” above or below connected by an arrow.) The player may also write the Solution, with a “0” or a “2” in it, and elsewhere on the paper indicate 0 = 2.

ET19 The Challenge-Scoring Rule of the basic game is as follows:

If there has been a challenge, then

(a) the Third Party Joiner(J) must join either the Challenger (C) or the Mover (M), and
(b) if the player that J joins has the burden of proving the existence of a Solution, then J must sustain the same burden of proof by independently writing a Solution, and
(c) if J is Correct, then J scores 8 if he has joined C and 10 if he has joined M, and
(d) C scores 10 if C is Correct, and
(e) M scores 10 if M is Correct, and
(f) if anyone is incorrect, he scores 6.

ET20 Whenever a player believes on his turn to move that

l. the previous move allows a Solution to be built with one more cube from Resources, but
2. the previous move is not a Flub because the allowing of such a Solution could not be avoided, then
3. he should say “forceout,” and
4. if no opponent challenges that forceout declaration within one minute (see ET22), each player should try to write a Solution on paper, using at most one cube from Resources.

ET21 The Non-Challenge Scoring Rule of the basic game is revised as follows.

If a player has declared forceout and no opponent has challenged that declaration, then

(a) each player who writes a correct Solution within two minutes scores 8;

(b) all other players score 6.

ET22 If a player (whose turn it is) calls "forceout," an opponent may challenge that declaration. The Challenger must specify reason A or reason B for challenging the forceout declaration.

(A) The forceout declarer should have challenged A or CA (that is, a previous Mover made a Solution possible with one more cube when that Mover could have avoided doing so). The Challenger (and the Third Party Joiner if agreeing with the Challenger) must write a Solution using at most one cube from those that were in Resources when the original A-Flub was committed and also must write an avoid move that could have been made instead of the original Flub.

(B) A Solution cannot be written with only one more cube. In this case, the Challenger (and the Third Party Joiner if agreeing with the Challenger) do nothing; the forceout declarer (and the Third Party Joiner if agreeing with the forceout declarer) must write a Solution using at most one cube from Resources.

For both situations (a) and (b), the forceout declaration must be challenged within the first minute of the two minutes allotted for writing Solutions after forceout has been called. If a Challenge is issued within this time and there is a Third Party, the Third Party has one minute to take a side on the Challenge. Then anyone writing a Solution has one additional minute to complete the Burden of Proof. (In a two party match delete the one minute for the Third Party to take a side. In this case, after the Challenge is issued, the Solution-writer has the remainder of the two minutes originally allotted after the forceout declaration for completing the Burden of Proof.)

In either case (a) or (b), if the forceout declaration is challenged, follow the Challenge-Scoring rule in ETl9.

ET23 Tournament Rounds will be scored in the following ways:

Three-person Games	
First place	6 points
Two-way tie for first	5 points
Three-way tie for first	4 points
Second place	4 points
Tie for second	3 points
Third place	2 points
Did not play	0 points

Two-person Games	
First place	6 points
Tie for first	5 points
Second place	4 points
Did not play	0 points

ET24 When a Tournament Round ends, each player must sign (or initial) the score sheet and the winner (or one of the winners) turns in the score sheet. If a player signs or initials a scoresheet on which his score is listed incorrectly and there is evidence that there was intent to deceive and the error was not a simple oversight, then

(a) if the error gives the player a lower score than that to which the player was entitled, the player shall receive the lower score;

(b) if the error gives the player a higher score than that to which the player was entitled, the player shall receive 0 points for that round.

ET25 Certain forms of behavior interfere with play and annoy or even intimidate opponents. Some examples are constant tapping on the table, humming or singing, loud or rude language, and constantly touching or moving the challenge block. If a player is guilty of such conduct, a judge will warn the player to discontinue the offensive behavior. After issuing this warning, the judge should inform the official in charge of that division and also the warned student's teacher (if available). Thereafter during that round or subsequent rounds, if the player again behaves in an offensive manner, a three-judge panel will consider the situation and may penalize the student one point for each violation after the warning. This panel will consist of the judge who issued the original warning, the chief judge of the division, and the student's teacher (with other judges filling these three positions if anyone of those listed is unavailable or if, for example, the judge who issued the warning and the chief judge are the same person). Flagrant misconduct or continued misbehavior may cause the player's disqualification by the panel for that round or the entire tournament.

➔**ET26** Two different symbols may be used to represent exponentiation in Goals and Solutions, * and ^. The only acceptable orientation of the ^ is upward, not sideways or pointing down.

MICHIGAN LEAGUE
OF ACADEMIC GAMES
OFFICIAL ADVENTUROUS RULES FOR
ON-SETS 2011-2012

ST1 The OFFICIAL RULES FOR BASIC ON-SETS (in the manual accompanying the game) will be used on all levels.

ST2 With appropriate adjustments for On-Sets, the following Equations rules will be in force: ET2, ET5, ET7, ET8 (second paragraph), ET9, ETll, ETl4, ET15, ET16, ETI7, ET19, ET20, ET21, ET22, ET23, ET24, and ET25. ET10 is also in force with the exception that each player may begin the round with sheet(s) containing preprinted Universe charts. (See Appendix A.)

ST3 The Advanced Game (with Restrictions) will be played at the Middle, Junior, and Senior levels. All levels will include variations (Adventurous On- Sets--see ST7). Elementary Division will play Adventurous On-Sets (without Restrictions) with variations. In Elementary the Goal-setter, before rolling the cubes, will first set out either two V and one /\ or one V and two /\ . In this way = and ⊆ will not appear in the Resources.

ST4 Rule ET4 also applies to On-Sets with the exception that during the first minute the cards are dealt and the cubes are rolled (in either order).

ST5 On each shake at least six but no more than 12 cards must be dealt, except in Senior Division where 10 to 14 cards (inclusive) must be dealt.

ST6 After the cards are dealt and placed in row(s) immediately next to each other in an easily seen pattern to all players, no player may touch them or in any way obstruct the other players' view of them until Solutions are checked. (See ET25.)

ST7 ADVENTUROUS VARIATIONS

I. ELEMENTARY VARIATIONS (grade 6 and below)

<u>No Forbidden</u> No player may play a non-digit cube to Forbidden.

> Note: Any digit cube not used in the Goal must be placed in Forbidden. (cf. Basic Rule 3.1 in the ON-SETS Manual in the game.)

<u>Required cube</u> The Solution must contain a _____cube. The player selecting this variation specifies when announcing the variation choice which non-digit symbol from the Resources fills the blank in the previous sentence.

> Note: If required – is called along with B wild, a B cube used as – does <u>not</u> satisfy the required cube variation.

<u>Wild cube</u> The ____ cube may vary and represent any symbol on the cubes except a digit. The _____ cube must stand for the same symbol everywhere it occurs in the Solution. The player selecting this variation specifies when announcing the variation which cube from the Resources varies for the shake. The cube that varies may not be a digit.

Notes:

a. If both B wild and B required are chosen, then a B cube must be in the Solution but may stand for another symbol.

b. In a Solution, if the wild cube stands for something other than itself, this fact must be indicated in writing in a clear and unambiguous manner. Follow the suggestions in ET18 with appropriate revision for On-Sets. For example, if B is wild but used as B, then this need not be indicated in writing. (See ST12.)

<u>∪ and ∩ interchangeable</u> In a Solution any ∪ may represent U or ∩ , and any ∩ may represent ∩ or ∪.

Notes:

a. ∪ and ∩ need not be used consistently. In a Solution one ∪ (or ∩) may be used as a ∪ and another ∪ (or ∩) used as ∩.

b. Any wild cube used as ∪ or ∩, gains the full interchangeable power granted ∪ and ∩ in this variation.

c. If ∪ (or ∩) wild and ∪-∩ interchangeable are both chosen for a shake, then, if ∪ (or ∩) is used just for itself or ∩, it need not be used consistently. However, if ∪ (or ∩) is used for any symbol other than ∪ or ∩, then it must represent that same symbol throughout the Solution.

d. Since this variation makes ∪ and ∩ "wild" in only a limited way, players are not required to indicate in writing where in the Solution a ∪ stands for ∩ or a ∩ stands for ∪. They should simply write the symbol they want mathematically. Thus, alternatives like those in ET18 are not required for this variation.

V and ∧ interchangeable In a Solution any V may represent V or ∧, and any /\ may represent /\ or V.

Note: The notes for ∪ and ∩ interchangeable, substituting V for ∪ and /\ for ∩, apply here.

Two operations The Solution must contain at least two operation symbols. The operation symbols are ∪, ∩,–, and ′.

Note: If a wild cube is also chosen, a wild cube used as an operation counts as an operation symbol.

Multiple operations Every operation sign in Resources, Required or Permitted may be used many times in a Solution. (Of course, each operation sign in Required must be used at least once in a Solution.)

Notes:

a. On an A-Flub Challenge, CA-Flub Challenge, or forceout, the "one more cube" in Resources is equivalent to a cube in Permitted. Therefore, if the cube from Resources is an operation cube, it may be used arbitrarily many times in a Solution.

b. On a P-Flub Challenge (Solution impossible), all cubes remaining in Resources are equivalent to Permitted cubes. Therefore, any operation sign remaining in

Resources when a P-Flub Challenge is made may be used many times in a Solution.

c. With this variation an operation cube is not being used to represent another symbol. Therefore, in their Solutions players may simply write an operation sign multiple times without any additional comment. The suggestions in ET18 do <u>not</u> apply to this variation.

* *

II. MIDDLE VARIATIONS (grades 7-8)

<u>No Forbidden</u> No player may play a non-digit cube to Forbidden.

Note: See the note following the Elementary No Forbidden variation.

<u>Required cube</u> The Solution must contain a ____ cube. The player selecting this variation specifies when announcing the variation which non-digit symbol from the Resources fills the blank in the previous sentence.

Notes:

a. See the note following the Elementary variation.

b. If a player selects = or $\subseteq$ required, this variation is satisfied by using the required cube in a Restriction. If the required cube is a color, V or Λ, or an operation symbol, the variation is satisfied by using that symbol in either a Restriction or the Set-Name. However, in the latter case, if the required symbol is played to Required, then, as usual, it must be used in both a Restriction (if one is made) and the Set-Name.

<u>Wild cube</u> The _____ cube may vary and represent any symbol on the cubes except a digit. The ___ must stand for the same symbol everywhere it occurs (Restriction(s)and Set-Name). The player selecting this variation specifies when announcing the variation choice which cube from the Resources varies for the shake. The cube that varies may not be =, $\subseteq$, or a digit.

Notes: See the notes following the Elementary variation.

∪ and ∩ interchangeable In a Solution any ∪ may represent ∪ or ∩ , and any ∩ may represent ∩ or ∪.

Notes: See the notes following the Elementary variation.

Null Restrictions Not Allowed Each Restriction must remove at least one card from the Universe. In a “chain” Restriction (see ST11) this variation is satisfied if any part of the chain removes a card.

V and Λ interchangeable In a Solution any V may represent V or Λ , and any /\ may represent /\ or V.

Note: See the note following the Elementary variation.

Two operations The Set-Name of the Solution must contain at least two operation symbols. The operation symbols are ∪, ∩, −, and ′.

Note: See the note following the Elementary variation.

Multiple operations Every operation sign in Resources, Required or Permitted may be used many times in a Solution (Set-Name or Restriction or both). (Of course, each operation sign in Required must be used at least once in the Set-Name and at least once in a Restriction if one is made.)

Notes: See the notes following the Elementary variation.

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III. JUNIOR VARIATIONS (grades 9-10)

Junior players may select any Middle Division variation or the following ones.

Shift From Permitted A player may on his/her move select a cube which has been played to Permitted and ‘shift’ it from Permitted to either Required or Forbidden.

Double color In the Universe each card containing the color will count double. The player selecting this variation specifies when announcing the variation choice which one of the four colors counts double.

* *

IV. SENIOR DIVISION (grades 11-12)

SPECIAL RULE FOR SENIOR DIVISION: The multiple operations variation is in effect for all shakes.

Senior Division players may select any of the Junior variations (except multiple operations, which is automatically in effect) or any of the following.

Blank card wild If the blank card has been dealt, a Solution-writer must specify in writing which colors, if any, are on it.

> Note: Suppose double color and blank card wild are both chosen with, say, B the double color. If a player chooses to put a B dot on the blank card, the blank card counts double for that player's Solution.

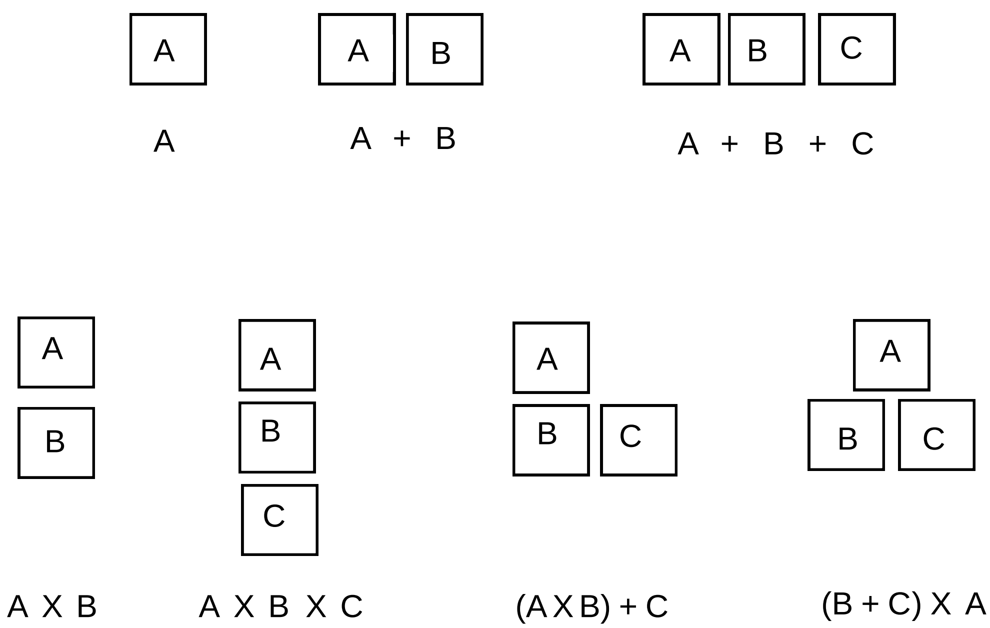
Symmetric difference The $-$ cube means "symmetric difference," that is, $A - B$ is interpreted as $(A - B) \cup (B - A)$, where these last two $-$ signs are the usual set subtraction.

> Notes:
> a. If a wild cube is also chosen with some symbol other than $-$ the wild cube, any wild cube used as $-$ represents the usual set subtraction since the $-$ cube means symmetric difference.
> b. If $-$ wild has already been selected for the shake, no player may select symmetric difference for that shake. Similarly, if symmetric difference has been chosen, no player may select $-$wild. In either case, according to ET4 the second player is assessed a one-point penalty. If both are chosen and no one objects, symmetric difference takes precedence.

c. In Solutions players may simply write the – sign with the understanding that it stands for symmetric difference. In accordance with note a, however, if a wild cube is representing the usual set subtraction, this fact needs to be indicated in writing.

* *

ST8 The following is a list of all possible legal Goals. Setting any other configuration of the digit cubes is a P-Flub. In each Goal below, except the first, an upside-down cube signifies the negative of that number.



ST9 No = or ⊆ cubes may be played to Forbidden. (V and /\ cubes may be played to Forbidden.)

STl0 On-Sets has a limited hierarchy of operations, as follows: in the absence of symbols of grouping, the unary operation ′ takes priority over the binary operations (∪, ∩, –, and special operations defined in variations). There is no hierarchy among the binary operations. See ET16 for the necessity of writing symbols of grouping before presenting a Solution to an opponent.

Examples:

a. For $R \cup G'$, take G' first, then union it with R. Thus $R \cup G' = G' \cup R$ but $R \cup G' \neq (R \cup G)'$.
b. $R \cup G \cap B$ is ambiguous. If one of the interpretations, $(R \cup G) \cap B$ or $R \cup (G \cap B)$, does not yield a set whose number of elements equals the Goal, the Solution is incorrect. The player presenting this Solution may not orally declare how it is to be interpreted.

ST11 Restrictions of the following form are permitted.

$X \subseteq Y \subseteq Z$ (abbreviating $X \subseteq Y$ and $Y \subseteq Z$)
$X = Y = Z$ (abbreviating $X = Y$ and $Y = Z$)
$X \subseteq Y = Z$ (abbreviating $X \subseteq Y$ and $Y = Z$)
$X = Y \subseteq Z$ (abbreviating $X = Y$ and $Y \subseteq Z$)

Restrictions of the following form are also permitted, where each one is worked out from left to right like those above.

$W \subseteq X \subseteq Y \subseteq Z$, $W = X = Y = Z$,

$W \subseteq X \subseteq Y = Z$ $W = X \subseteq Y \subseteq Z$, etc.

CAUTION: A common error is putting parentheses around part of a chain Restriction, like this: $(X \subseteq Y) \subseteq Z$, $X = (Y = Z)$, etc. Such parentheses make the chain meaningless. However, this does not mean that parentheses may not be used at all in Restrictions. Parentheses may legitimately be placed within the left or the right side of an = or $\subseteq$ statement, as in these examples.

$(R \cup B) - G = V$, $B = (G \cup R)' \subseteq V$, $R' = B \subseteq (R - Y) \cup V$

Notice in these correct examples that the parentheses do not enclose an = or $\subseteq$ symbol.

ST12 Equations rule ETI8 is in force in On-Sets. For guidelines for indicating the meaning of symbols for specific variations, see Appendix A.